

Lecture 3_ Day Ahead market

Obj:

- explain the fundamentals of the day-ahead market
- Describe the concept of day-ahead market coupling in Europe
- Analyse the network effects in the market-clearing problem,
- Formulate the nodal and zonal day-ahead market-clearing optimisation problems
- Understand the concept of flow-based market coupling.

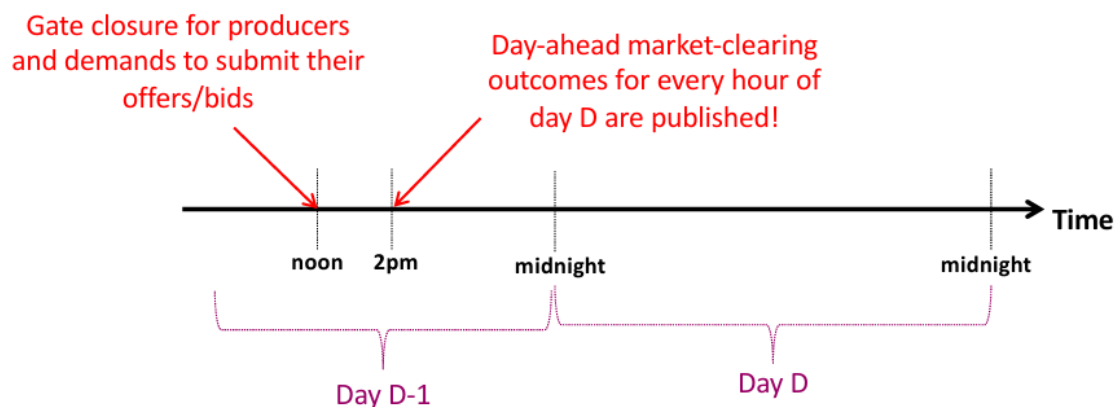
Outline

- Fundamentals of Day-Ahead Market Clearing
- Single Day-Ahead Coupling (SDAC) in Europe
- Network effects on Market-Clearing Outcomes
- Power-Flow Equations
- Market-Clearing Optimisation Problem: Nodal vs Zonal
- Flow-Based Market Coupling in Europe.

1.Fundamentals of Day-Ahead Market Clearing

Day-ahead market in Nord Pool

Day-ahead market in Nord Pool



On day D-1, the day-ahead market clearing determines production and consumption quantities, along with **uniform** market-clearing prices, for **each hour** of day D. These quantities and prices may vary from hour to hour but remain fixed within each hour.

- Time resolution for the day-ahead market, also referred to as market time unit (MTU):
15-minute intervals starting **June 11, 2025**

Why go from 1h interval -> 15-minute intervals is good?

- Because the Renewable energy is a factor, RE is very intermittent.
New forecasting is better for 15 mins instead of 1h

- Have more service will join, for example battery.

Day-ahead market: Why?

Discussion:

Why is a day-ahead market necessary? Why not wait until real-time (or closer to real-time) to clear the electricity market instead?

- Operational reasons: Many conventional generation units, such as nuclear and coal plants, are slow and inflexible -> they have various operational constraints and require advance scheduling to operate efficiently
- Economic reasons: Economic studies, e.g., show that forward financial markets, any market cleared before real-time operation, enhance market competitiveness, and reduce market power (strategic bidding). this prevents market participants from manipulating market-clearing outcomes for their own benefit.

In future 100% renewable-based power systems with massive integration of distributed energy resources, day-ahead markets may not be necessary. Instead, long-term futures markets and near real-time operational markets might be sufficient.

Alternatively, could “electricity as a service” be the solution?

What is the service market: Internet server (pay for the capacity)

2. Single Day-Ahead Coupling (SDAC) in Europe

Single Day-ahead Coupling (SDAC) in Europe

It is better to go 5 separated zone markets or just one market?

- One market for everything is better in terms of resource attribution
- SDAC Launched in February 2014

SDAC: Single Day-Ahead Coupling

Source: ENTSO-E [\[link\]](#)



“ The aim of Single Day-ahead Coupling (SDAC) is to create a *single pan European cross zonal* day-ahead electricity market. SDAC allocates scarce cross-border transmission capacity in the most efficient way by coupling wholesale electricity markets from different regions through a common algorithm, simultaneously taking into account *cross-border transmission constraints*, and thereby *maximizing social welfare*. ”



“ An integrated day-ahead market increases the overall efficiency of trading by:

- Promoting *effective* competition
- Increasing *liquidity*
- Enabling a *more efficient utilization* of the generation resources across Europe. ”

In SDAC, increasing liquidity means enlarging the volume and diversity of bids/offers in the day-ahead market, making trading more efficient, competitive, and stable across Europe.

Liquidity is also competition.

Involved Parties:

SDAC: Single Day-Ahead Coupling

Source: ENTSO-E [\[link\]](#)



Involved parties

Transmission System Operators (TSOs):

50Hertz Transmission, ADMIE, Amprion, APG, AST, ČEPS, Creos, EirGrid, Elering, ELES, ELIA, Energinet, ESO, Fingrid, HOPS, Litgrid, MAVIR, PSE, REE, REN, RTE, SEPS, SONI, Statnett, Svenska Kraftnät, TenneT DE, TenneT NL, Terna, Transeletrica, and TransnetBW.



Nominated Electricity Market Operators (NEMOs):

BRM, BSP, CROPEX, SEMOpx (EirGrid and SONI), EPEX, EXAA, GME, HEnEx, HUPX, IBEX, Nord Pool, OMIE, OKTE, OPCOM, OTE, and TGE.

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How does SDAC work?

Pan-European Hybrid Electricity Market Integration Algorithm (EUPHOMIA)

Is an algorithm developed to couple all different market rules, to calculate electricity prices across Europe, and to allocate cross-border capacity on a day-ahead basis.

- Day-ahead market

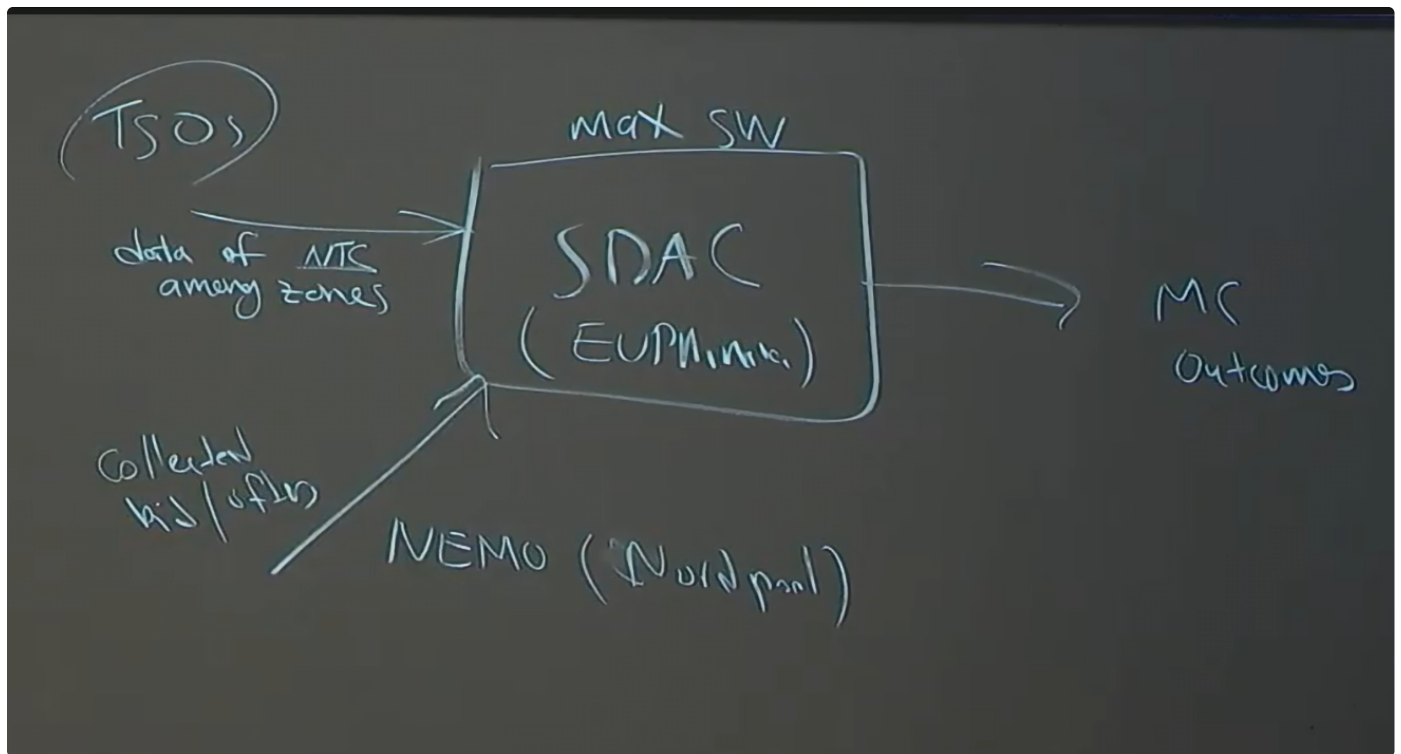
Question: Is Nordpool the one who clear the market or EUPHOMIA clear the market for whole Europe and Nordic?

NTC: Net Transfer Capacity

NEMO: A **Nominated Electricity Market Operator (NEMO)** is a power exchange or entity designated by national regulatory authorities in European Union member states to run integrated wholesale day-ahead and intraday electricity markets

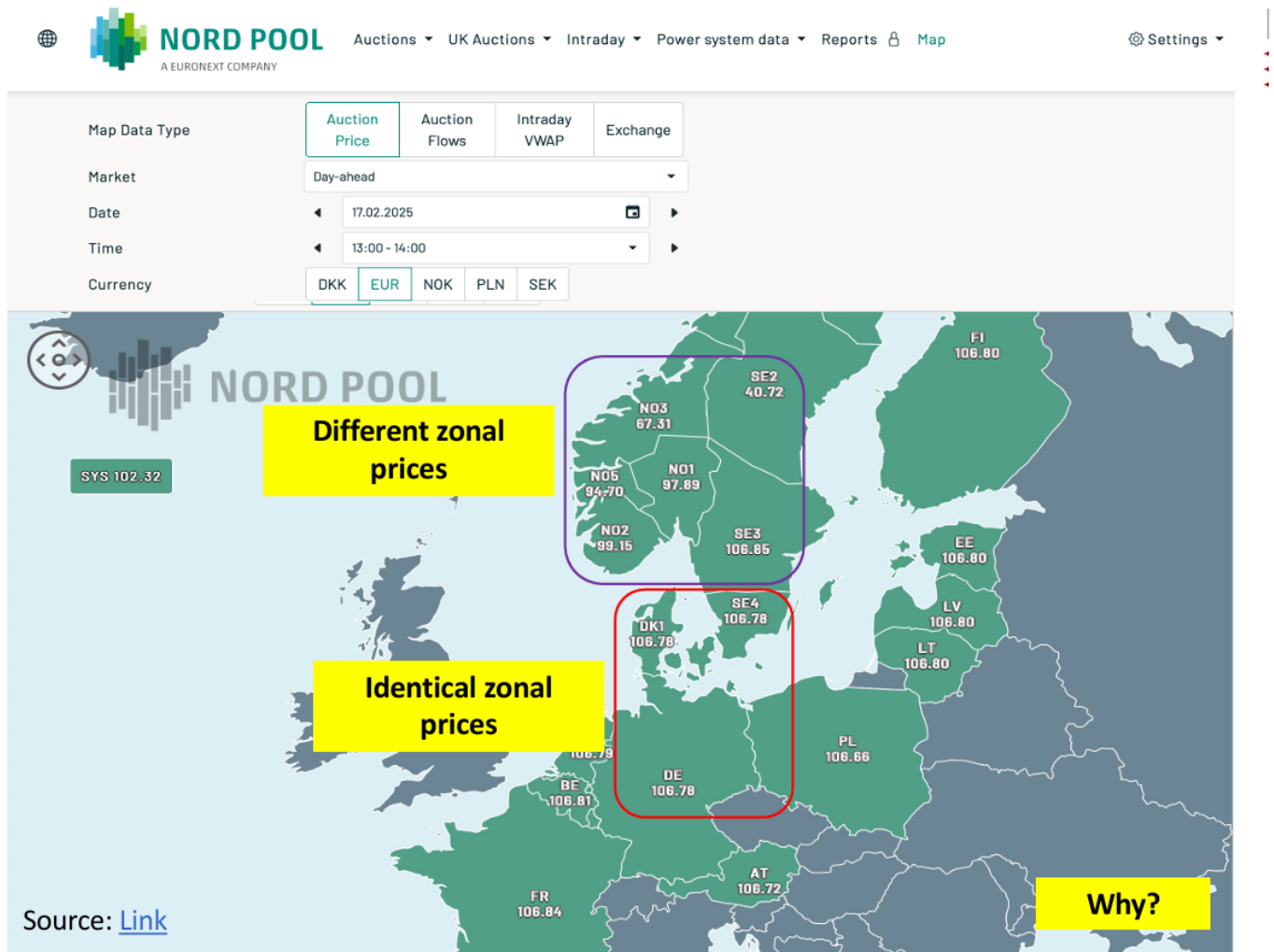
Nord Pool: Bid, offer, collector -> put all the bid offer in the same place to EUPHOMIA

-> therefore, EUPHOMIA clear the market



SYS : System Price: The market clearing price (in Euro/MWh) assuming no transmission grid constraints (copper-plate system)

Feature	Copper-plate (System Price)	Real SDAC (Zonal Prices)
Price outcome	One single European price	Different prices per bidding zone
Grid constraints	Ignores all transmission limits	Transmission constraints fully considered
Cross-border flows	Assumed infinite capacity	Limited by ATC / FB capacities
Dispatch logic	Pure economic dispatch	Constrained dispatch respecting grid limits
Drivers of price	Only supply & demand curves	Supply, demand + congestion + network limits
Congestion	No congestion possible	Congestion creates price differences
Algorithm behavior	Ideal unconstrained solution	EUPHEMIA computes feasible zonal prices
Interpretation	"What price would be if grid was perfect?"	"Actual market price given real grid bottlenecks"



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To answer the question why they have different zonal price and identical prices, it happens because of the transmission limit

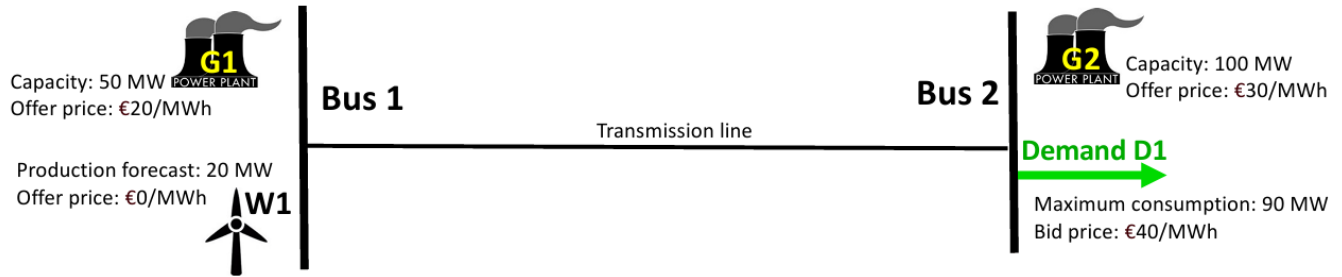
3. Network effects on Market-Clearing Outcomes

Bus: Power system term

Node: Graph

An illustrative example

Let us consider this two-bus power network with three producers (W1, G1, and G2) and a single demand (D1):



For the case where the transmission line capacity is **80 MW**, clear the market:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **50**
- Production level (MW) of G2: **20**

- Power flow from Bus 1 to Bus 2 (MW): **70**

- Market-clearing price (€/MWh) at Bus 1: **30**
- Market-clearing price (€/MWh) at Bus 2: **30**

- Payment to W1 (€): $20 \times 30 = 600$
- Payment to G1 (€): $50 \times 30 = 1500$
- Payment to G2 (€): $20 \times 30 = 600$
- Payment by D1 (€): $90 \times 30 = 2700$

Budget Balance:
Total payment to W1+G1+G2
= Payment by D1

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Since any 1 MW change in Demand (at either bus) result in a 30 change in the social welfare of the system

What happens if demand increases by 1 MW?

If demand goes from 90 → 91 MW:

- W1 is already full (20 MW)
- G1 is already full (50 MW)
- So the **next MW must come from G2**, at **€30/MWh**

The system gains **€40** of value (because demand is willing to pay 40 €/MWh) but loses **€30** of cost (because G2 costs 30 €/MWh).

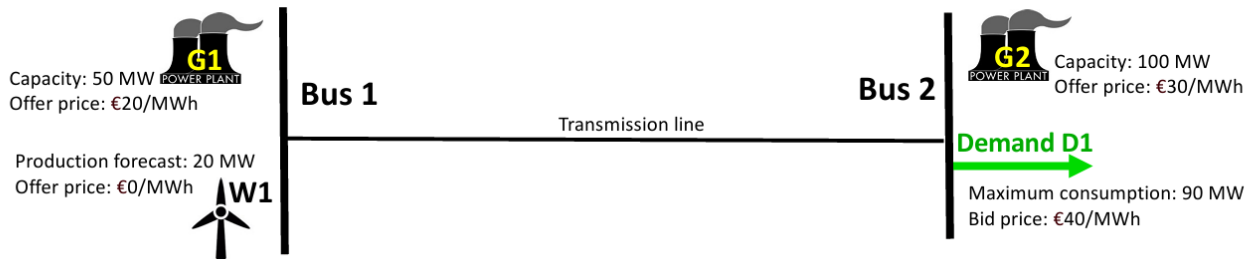
So the **net social welfare change** is:

$\Delta SW = 40 - 30 = 10 \text{ €/MW}$ But the **marginal cost** is **€30**, and that is what sets the price.

An illustrative example



Let us consider this two-bus power network with three producers (W1, G1, and G2) and a single demand (D1):



Now, we reduce the transmission line capacity to **60 MW**:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **40**
- Production level (MW) of G2: **30**
- Power flow from Bus 1 to Bus 2 (MW): **60 (congested)**
- Market-clearing price (€/MWh) at Bus 1: **20**
- Market-clearing price (€/MWh) at Bus 2: **30**

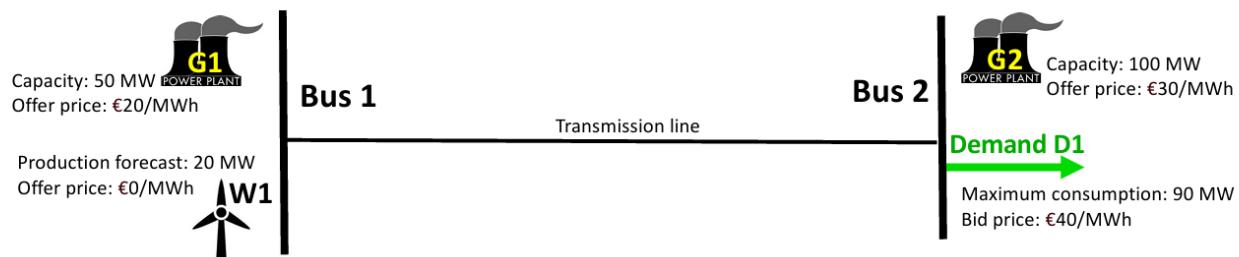
Why different prices?

Since any 1 MW change in demand at bus 1 causes a €20 change in the social welfare of the system, while the same change at bus 2 results in a €30 change.

An illustrative example



Let us consider this two-bus power network with three producers (W1, G1, and G2) and a single demand (D1):



Now, we reduce the transmission line capacity to **60 MW**:

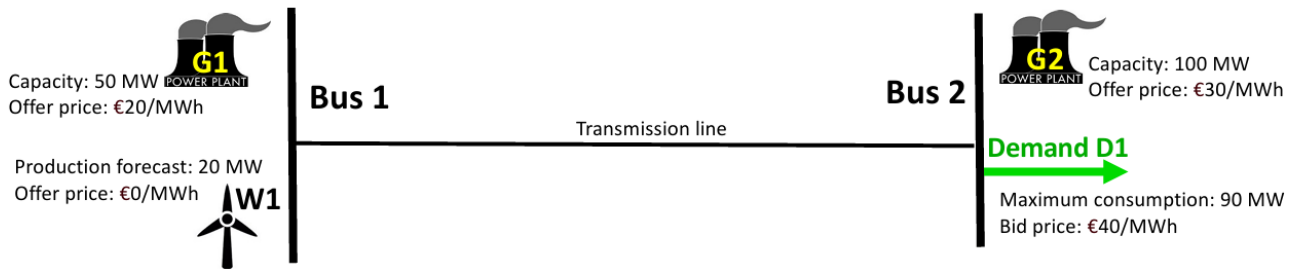
- Production level (MW) of W1: **20**
- Production level (MW) of G1: **40**
- Production level (MW) of G2: **30**
- Power flow from Bus 1 to Bus 2 (MW): **60 (congested)**
- Market-clearing price (€/MWh) at Bus 1: **20**
- Market-clearing price (€/MWh) at Bus 2: **30**

- Payment to W1 (€): $20 \times 20 = 400$
- Payment to G1 (€): $40 \times 20 = 800$
- Payment to G2 (€): $30 \times 30 = 900$
- Payment by D1 (€): $90 \times 30 = 2700$

- Total payment to W1+G1+G2
= $400 + 800 + 900 = \text{€}2100$
- Payment by D1 = **€2700**

An illustrative example

Let us consider this two-bus power network with three producers (W1, G1, and G2) and a single demand (D1):



Now, we reduce the transmission line capacity to **60 MW**:

Who is receiving the €600 surplus, and what does it represent?

It is called the “**congestion rent**”, also referred to as “**congestion revenue**”:

$$= \text{Flow} \times [\text{price at bus 2} - \text{price at bus 1}]$$

$$= 60 \times [30 - 20] = 600.$$

This surplus belongs to the grid owner (the TSO in this case). One can think of the grid owner as a “**spatial arbitrager**”, buying 60 MW of power at bus 1 at the lower price of €20/MWh, transferring it to bus 2, and then selling it at the higher price of €30/MWh. This is similar to how storage acts as a **temporal arbitrager**.

Congestion rent

- when the transmission limit is 80 MW (**uncongested case**):
 $= 80 \times [30 - 30] = €0.$
- when the transmission limit is 60 MW (**congested case**):
 $= 60 \times [30 - 20] = €600.$

Does this suggest that the TSO is a "for-profit" entity that earns revenue from congestion?

No, the TSO is not for-profit. Congestion revenue exists, but it must be reinvested into the grid or used to reduce tariffs. It cannot be kept as profit.

TSOs are *regulated, non-profit* entities

In Europe (ENTSO-E countries):

- TSOs **cannot** keep congestion revenue as profit
- TSOs **cannot** distribute it to shareholders
- TSOs **must** use it for regulated purposes

They are legally required to use congestion income for:

1. Maintaining or increasing cross-border capacity

(Upgrading lines, reinforcing transformers, improving stability)

2. Ensuring secure grid operation

(Reserve procurement, redispatch costs, remedial actions)

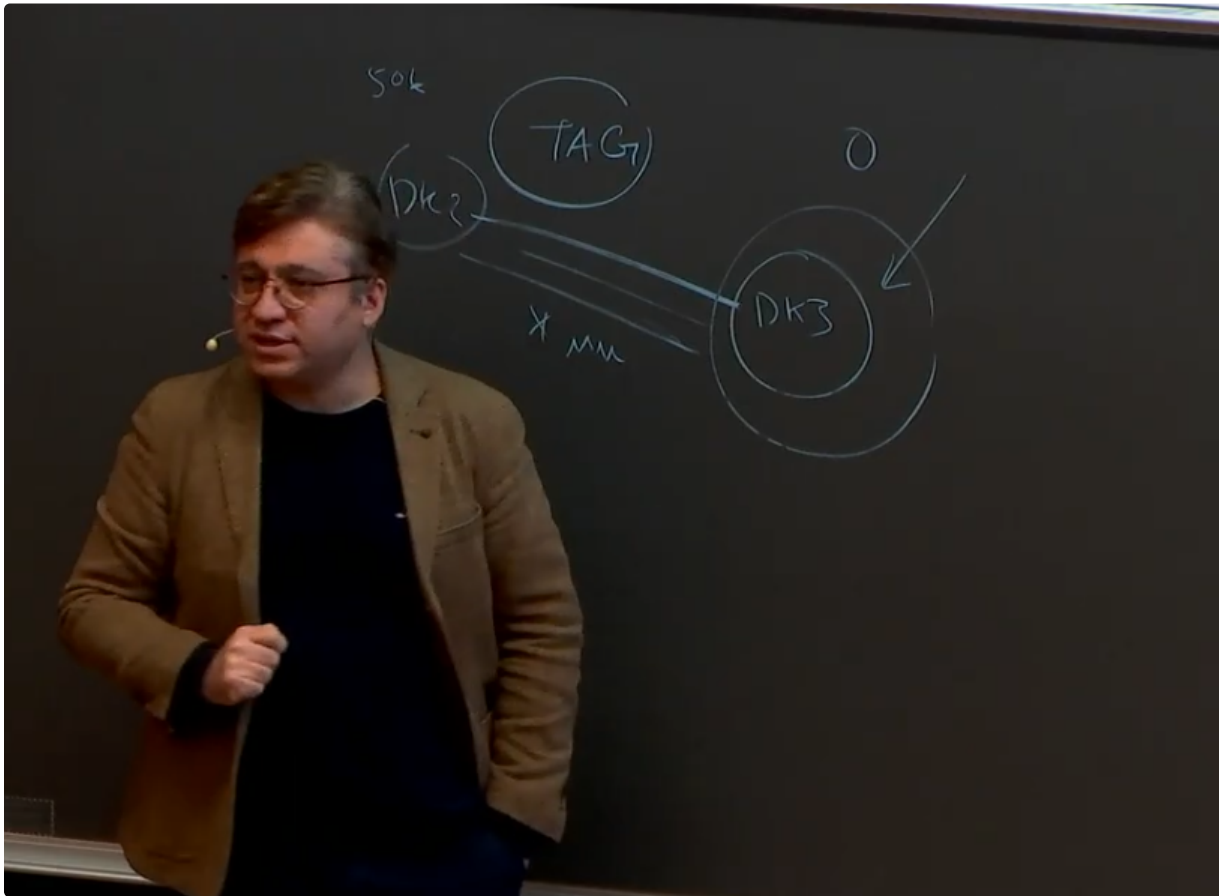
3. Reducing tariffs for consumers

(If there is surplus after covering grid costs)

This is written in EU Regulation 2019/943 (Electricity Regulation).

Possible Ms thesis: Arguing who's gonna invest transmissison line

TAG:



== Market Clearing as an Optimisation (with Network)

p 1 ->2: New primal variable: Power flow (in MW) from bus 1 to bus

Market clearing as an optimization (with network)



$$\text{Maximize}_{p^{D1}, p^{W1}, p^{G1}, p^{G2}, p^{1 \rightarrow 2}} 40 p^{D1} - 0 p^{W1} - 20 p^{G1} - 30 p^{G2}$$

subject to:

$$0 \leq p^{W1} \leq 20 \quad : \quad \underline{\mu}^{W1}, \bar{\mu}^{W1}$$

$$0 \leq p^{G1} \leq 50 \quad : \quad \underline{\mu}^{G1}, \bar{\mu}^{G1}$$

$$0 \leq p^{G2} \leq 100 \quad : \quad \underline{\mu}^{G2}, \bar{\mu}^{G2}$$

$$0 \leq p^{D1} \leq 90 \quad : \quad \underline{\mu}^{D1}, \bar{\mu}^{D1}$$

Question:

- Why is a lower bound (-60 MW) enforced?

$$-60 \leq p^{1 \rightarrow 2} \leq 60 \quad : \quad \underline{\mu}^{1 \rightarrow 2}, \bar{\mu}^{1 \rightarrow 2} \quad \text{Power flow limits}$$

$$p^{1 \rightarrow 2} - p^{W1} - p^{G1} = 0 \quad : \quad \lambda^{\text{Bus1}}$$

$$p^{D1} - p^{1 \rightarrow 2} - p^{G2} = 0 \quad : \quad \lambda^{\text{Bus2}}$$

Market clearing as an optimization (with network)



$$\text{Maximize}_{p^{D1}, p^{W1}, p^{G1}, p^{G2}, p^{1 \rightarrow 2}} 40 p^{D1} - 0 p^{W1} - 20 p^{G1} - 30 p^{G2}$$

subject to:

$$0 \leq p^{W1} \leq 20 \quad : \quad \underline{\mu}^{W1}, \bar{\mu}^{W1}$$

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$$0 \leq p^{D1} \leq 90 \quad : \quad \underline{\mu}^{D1}, \bar{\mu}^{D1}$$

$$-60 \leq p^{1 \rightarrow 2} \leq 60 \quad : \quad \underline{\mu}^{1 \rightarrow 2}, \bar{\mu}^{1 \rightarrow 2}$$

$$p^{1 \rightarrow 2} - p^{W1} - p^{G1} = 0 \quad : \quad \lambda^{\text{Bus1}}$$

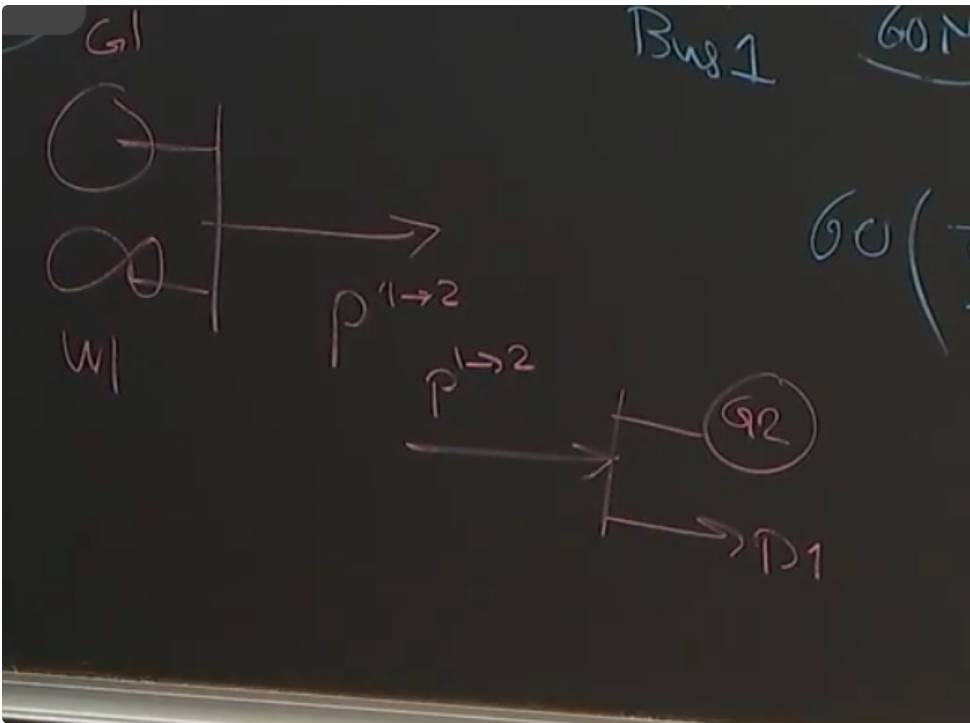
$$p^{D1} - p^{1 \rightarrow 2} - p^{G2} = 0 \quad : \quad \lambda^{\text{Bus2}}$$

Power balance equalities (one per bus)

Why first term p1-2 is positive but 2nd is negative?

Because G1 and G2 send power p1-2 to bus 2

And next that power + pG2 send to the demand



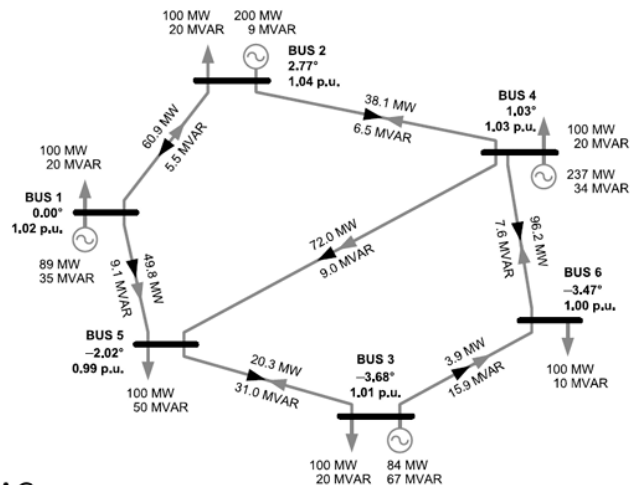
Power Flow Equations

Power flow

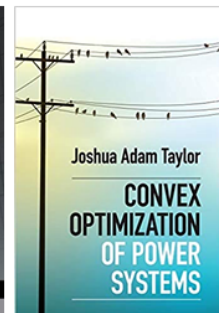
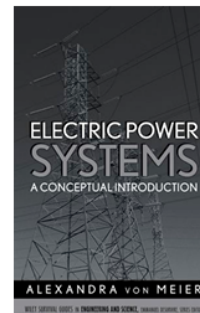


Remember that the power flow in an electrical transmission network is governed by the **laws of physics**, such as Kirchhoff's and Ohm's laws.

In fact, **AC power flow equations** are used to calculate the amount of power to be transferred across each transmission line.



- Are you familiar with or do you recall the AC power flow equations?
- Among many other relevant books, you can learn about AC power flow equations from these two books:



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AC power flow is the full, physics-accurate set of equations that describe how **real (P)** and **reactive (Q)** power move through an electrical network. It's exactly what your Lecture 3 PDF is referring to when it says:

AC power flow = the nonlinear equations that compute voltages, angles, and power flows in an AC grid, based on **Kirchhoff's laws**. They determine *how electricity actually flows* in real time.

Electricity in transmission grids is **alternating current (AC)**. This means every bus has:

- a **voltage magnitude** V_i
- a **voltage angle** θ_i

AC power flow equations: Quick review



For every bus $i = \{1, 2, \dots, n\}$, the following two equalities hold:

$$P_i = \sum_{k=1}^n |V_i| |V_k| [g_{ik} \cos(\theta_i - \theta_k) + b_{ik} \sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^n |V_i| |V_k| [g_{ik} \sin(\theta_i - \theta_k) - b_{ik} \cos(\theta_i - \theta_k)]$$

Definition of symbols:

- Generated/consumed **active power** at each bus i , denoted by P_i ,
- Generated/consumed **reactive power** at each bus i , denoted by Q_i ,
- **Voltage magnitude** at each bus i , denoted by V_i ,
- **Voltage angle** at each bus i , denoted by θ_i ,
- **Conductance** and **susceptance** of the line connecting buses i and k (input network data), denoted by $g_{i,k}$ and $b_{i,k}$.

Depending on the type of each bus i , two out of the four variables, P_i , Q_i , V_i and θ_i , are known, while the other two are unknown. This ensures that the number of unknown variables and equations is balanced.

These contain for the equation

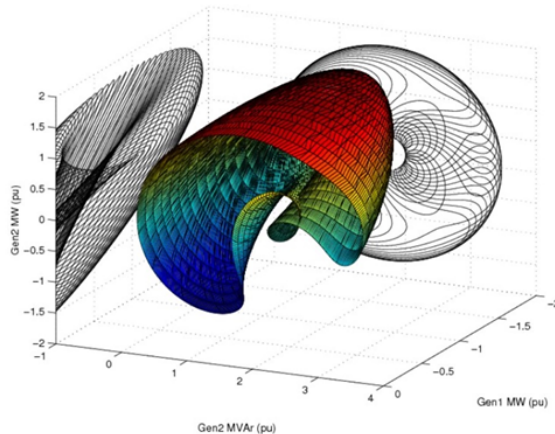
- **products** of variables
- **sine** and **cosine** terms
- **nonlinear coupling** between buses

This creates a feasible region that looks like the weird shape shown in the slide (the Hiskens figure) — not a nice convex polytope.

AC power flow equations: Quick review

However, the AC power flow equations are non-linear and non-convex, which adds complexity to solving them (NP-hard problem).

When incorporating these equations into an optimization problem, there is no guarantee of obtaining the global optimal solution.



An example of the feasible space in an AC optimal power flow problem (the feasible area for sample 3 variables limited by power flow constraints).

Credit: Ian Hiskens et al.

Non-convex problems can have:

- multiple local optima
- saddle points
- disconnected feasible regions

So even if you run a solver:

👉 You cannot guarantee it found the *global* optimum.

If SDAC or any day-ahead market tried to use **full AC power flow**, the market-clearing problem would:

- take too long
- sometimes fail to converge
- sometimes give suboptimal or inconsistent prices
- not guarantee welfare maximization

This is unacceptable for a market that must clear **Europe in 17 minutes**.

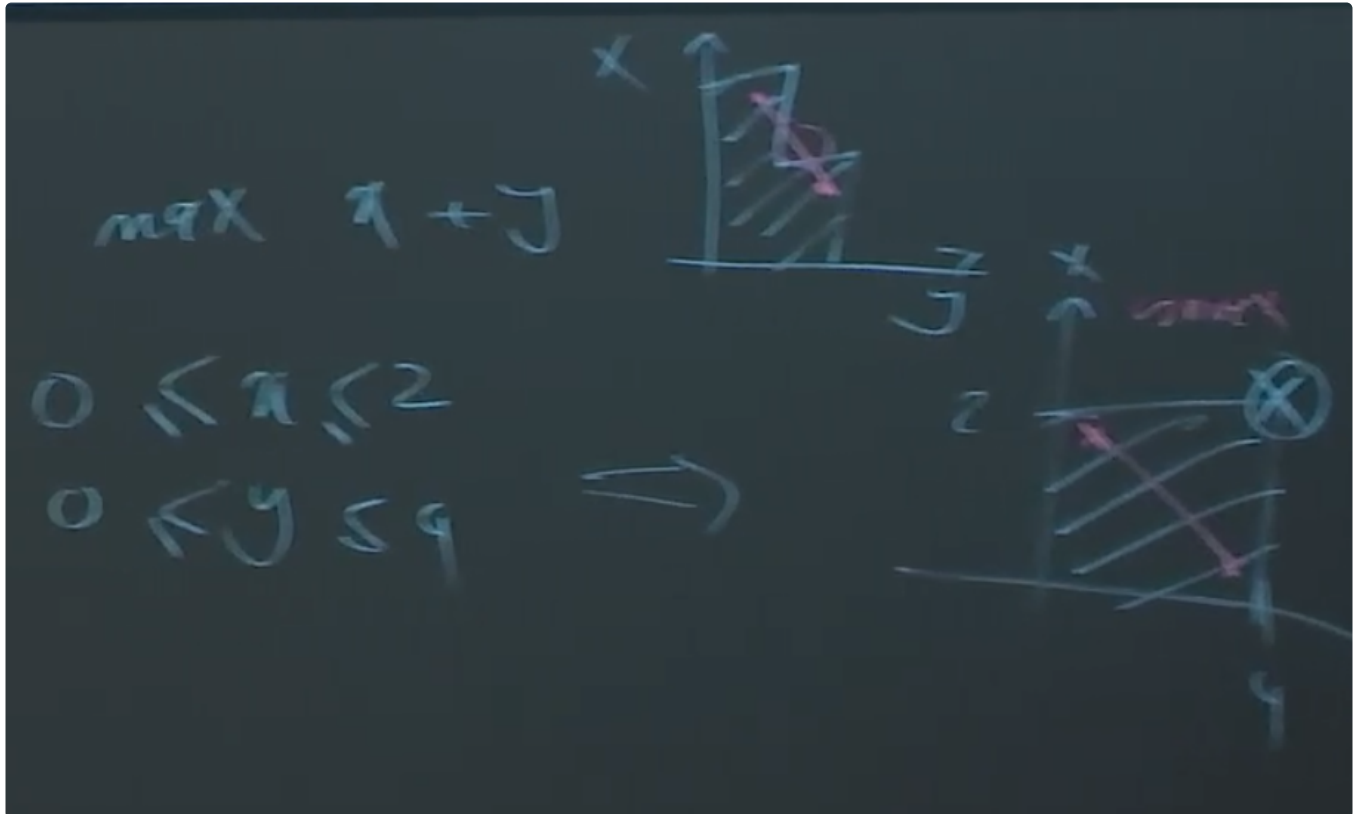
So instead:

👉 Markets use **linearized DC power flow**, which *is* convex

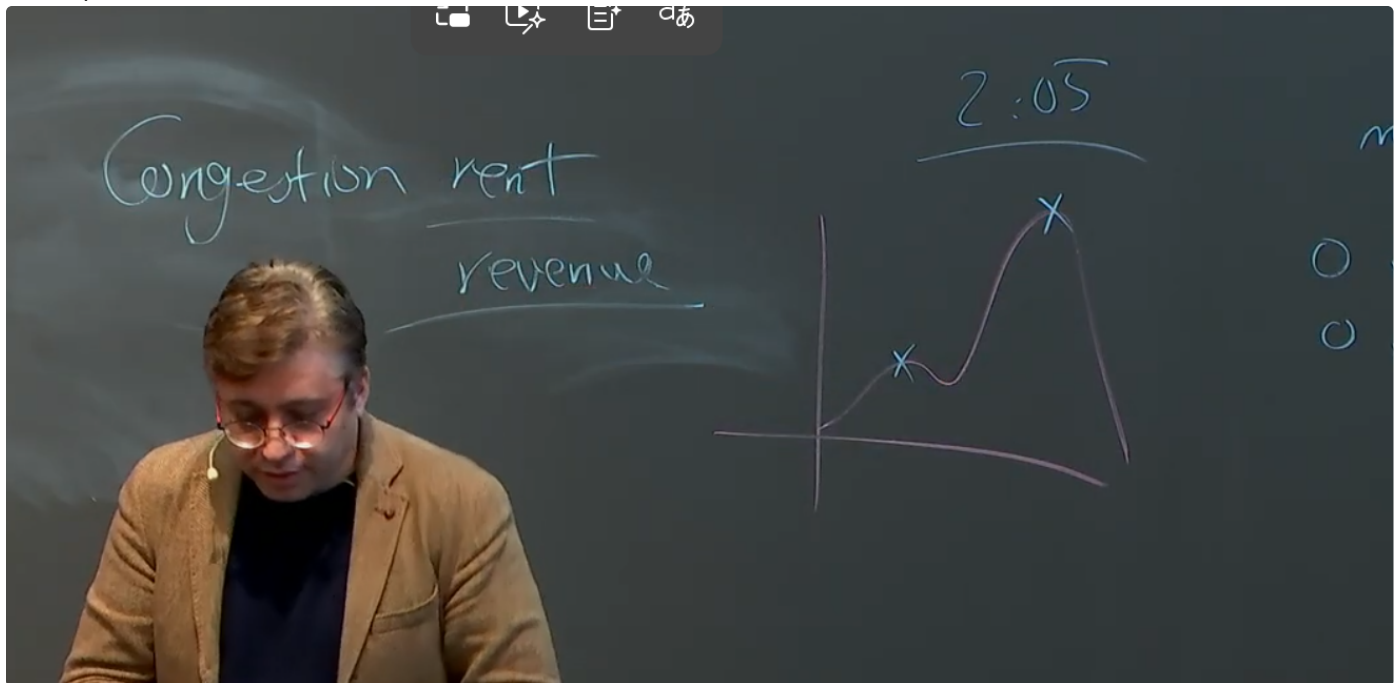
This makes the optimization:

- fast
- reliable
- globally optimal

- predictable



Local optimisation



Linearized AC power flow approximation

Also, referred to as “DC power flow approximation”:

$$P_i = \sum_{k=1}^n |V_i||V_k| [g_{ik} \cos(\theta_i - \theta_k) + b_{ik} \sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^n |V_i||V_k| [g_{ik} \sin(\theta_i - \theta_k) - b_{ik} \cos(\theta_i - \theta_k)]$$

↓

$$P_i = \sum_{k=1}^n b_{ik} (\theta_i - \theta_k)$$

Recall symbols:

- Generated/consumed **active power** at each bus i , denoted by P_i ,
- **Voltage angle** at each bus i , denoted by θ_i ,
- **susceptance** of the line connecting buses i and k (input network data), denoted by $b_{i,k}$.

Power flow equations in the market-clearing problem

Our recent example of the market-clearing optimization problem with two buses and one transmission line

Maximize $40 p^{D1} - 0 p^{W1} - 20 p^{G1} - 30 p^{G2}$
 subject to:

$$0 \leq p^{W1} \leq 20 : \underline{\mu}^{W1}, \bar{\mu}^{W1}$$

$$0 \leq p^{G1} \leq 50 : \underline{\mu}^{G1}, \bar{\mu}^{G1}$$

$$0 \leq p^{G2} \leq 100 : \underline{\mu}^{G2}, \bar{\mu}^{G2}$$

$$0 \leq p^{D1} \leq 90 : \underline{\mu}^{D1}, \bar{\mu}^{D1}$$

$$-60 \leq p^{1 \rightarrow 2} \leq 60 : \underline{\mu}^{1 \rightarrow 2}, \bar{\mu}^{1 \rightarrow 2}$$

$$p^{1 \rightarrow 2} - p^{W1} - p^{G1} = 0 : \lambda^{Bus1}$$

$$p^{D1} - p^{1 \rightarrow 2} - p^{G2} = 0 : \lambda^{Bus2}$$

Power flow constraints

Linearized power flow equations

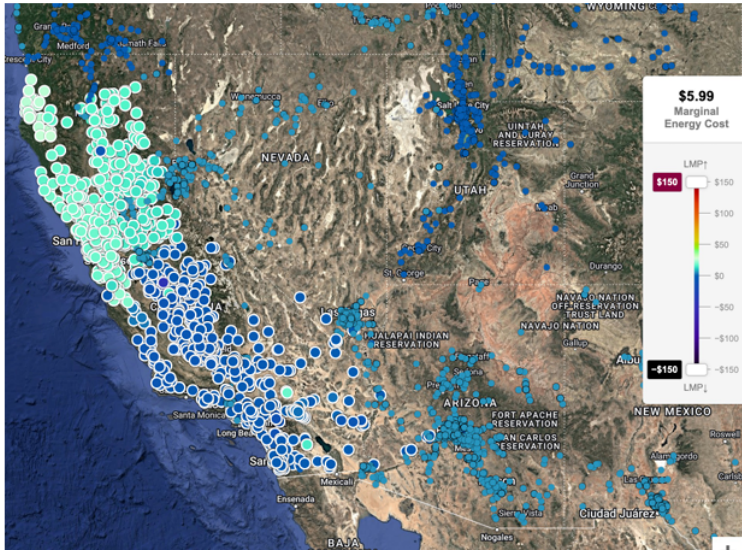
$$P_i = \sum_{k=1}^n b_{ik} (\theta_i - \theta_k)$$

4. market optimisation Nodal vs Zonal

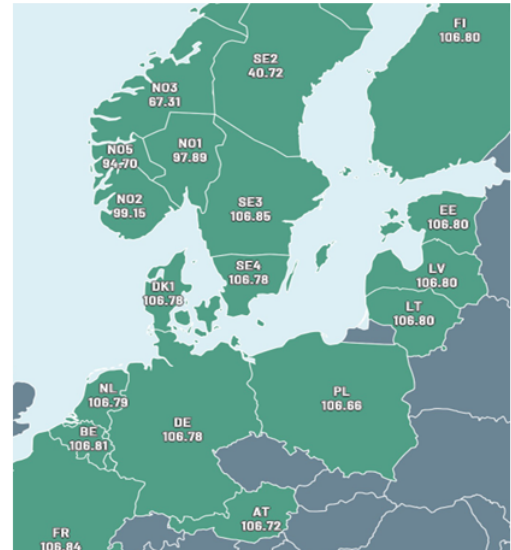
Nodal vs. zonal electricity markets



Nodal prices (in \$/MWh) in **CAISO** (California, U.S.)
February 16, 2025, 10-11pm ([link](#))



Zonal prices (in €/MWh) in **Europe**
February 17, 2025, 1-2pm ([link](#))



In California: Each different point has different zone

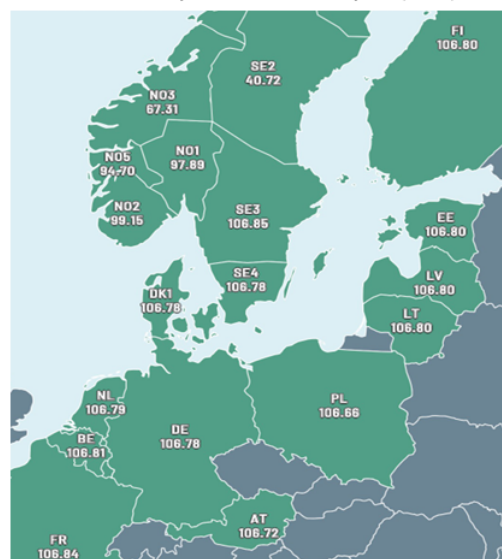
Nodal vs. zonal electricity markets



The limits of critical transmission lines are enforced to some extent in the “flow-based coupling” approach. This will be discussed in the next slides.

- Each bidding zone in Europe has a **single** price (e.g., one price for the entire Germany).
- Transmission line capacity limits **within** bidding zones are **not** enforced in the market-clearing optimization problem.
- Price differences between two zones occur only when power trade between them reaches the **net transfer capacity (NTC)** set by the TSOs.

Zonal prices (in €/MWh) in **Europe**
February 17, 2025, 1-2pm ([link](#))

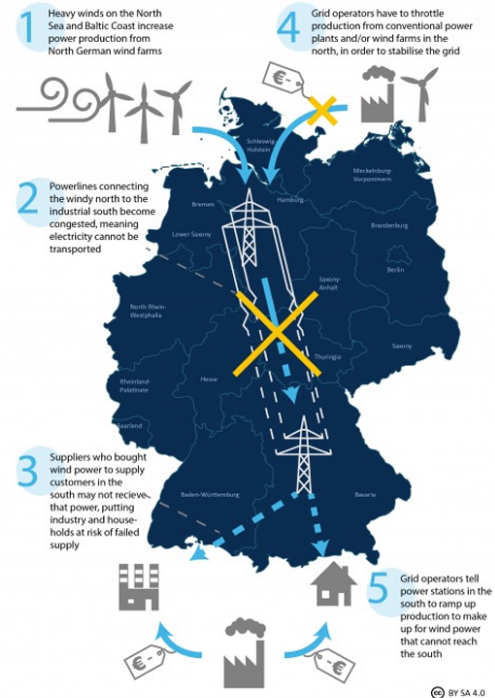


Feature	Nodal Market (e.g., PJM, CAISO – U.S.)	Zonal Market (Europe – SDAC)	Example from lecture
Price granularity	One price per node (bus)	One price per bidding zone	Germany = one price for entire country
Internal grid constraints	Fully enforced (all line limits included in optimization)	Ignored in market clearing (except some critical lines in flow-based)	Germany's north–south congestion NOT reflected in DA price
Cross-border constraints	Enforced	Enforced (NTC or flow-based)	Price differences only when NTC between zones is hit
Congestion visibility	Local congestion → local price differences	Only inter-zonal congestion → zonal price differences	SE2 ≠ SE3 ≠ SE4 in your screenshot
Market-clearing model	AC/DC OPF with full network	Welfare maximization + NTC constraints	EUPHEMIA uses DC + NTC/FB constraints
Efficiency	High (uses full physics)	Lower (internal redispatch needed)	Denmark redispatch due to internal bottlenecks
Who fixes internal congestion?	Market (prices reflect it)	TSO after market (redispatch)	Energinet redispatches DK1–DK2 flows
When do prices differ?	Whenever ANY line congests	Only when cross-border line congests	Germany–Denmark price split when interconnector is full
Example scenario	Line inside city congested → price difference between Bus A and Bus B	Internal line congested but ignored → same zonal price	DK1 & DK2 same price even if internal lines congested

Nodal vs. zonal electricity markets

Redispatch Cost:

- After market clearing, TSOs request every market participant with an accepted bid/offer to provide a detailed production/consumption plan within zones.
- TSOs then simulate whether market outcomes are **feasible** concerning transmission network limits.
- If infeasibilities arise, **redispatch** is required!
- German TSOs, in particular, incur **significant annual redispatch costs**—amounting to multiple billions of euros per year—due to expensive redispatch actions.



Source: Clean Energy Wire

DTU Wind, Technical University of Denmark

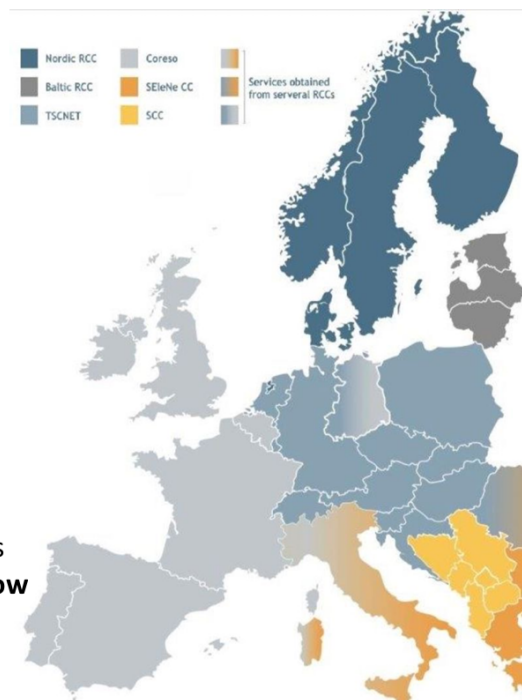
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Cross-border coordination in Europe

There are six Regional Coordination Centres (RCCs) across Europe:

- **Nordic RCC** – based in Copenhagen
- **TSCNET Services** – based in Munich
- **Coreso** – based in Brussels
- **SCC** – based in Serbia
- **Baltic RSC** – served from the three national control centres
- **Selene CC** – based in Greece

Among other tasks, RCCs coordinate with TSOs on **NTCs (Net Transfer Capacities)** or **power flow coupling parameters** to enhance cross-border electricity exchanges and grid stability.

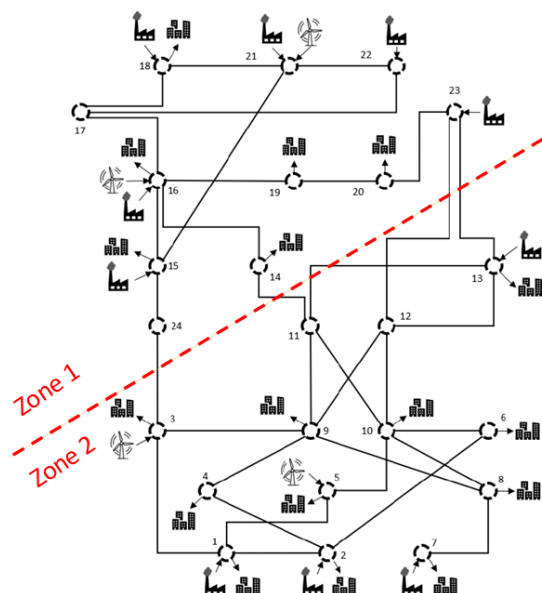


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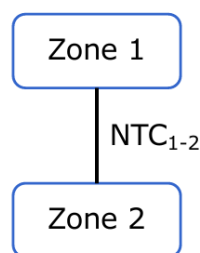
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Market-clearing optimization: Nodal vs. zonal

Example: A **zonal** system with **two** zones

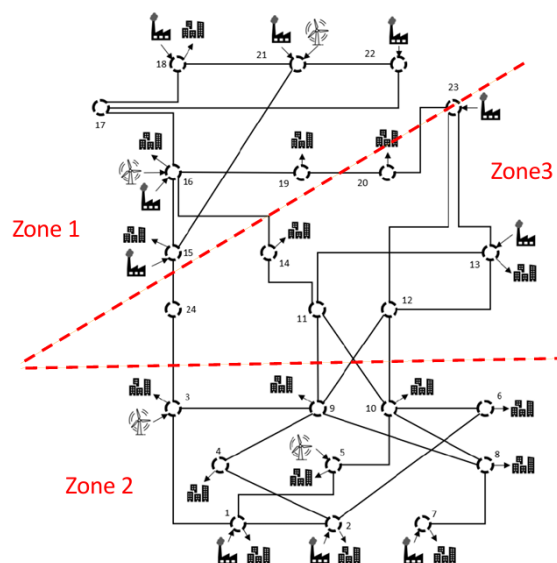


- Zone 1 includes nodes 1 to 13, while Zone 2 encompasses the remaining nodes.
- There is one NTC value between the two connecting zones.
- Transmission line limits within the zones will be disregarded.

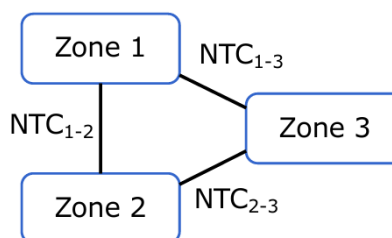


Market-clearing optimization: Nodal vs. zonal

Example: A **zonal** system with **three** zones



There are **three** NTC values, one for each pair of connecting zones.



NTC is the input data for market clear

NTC is determined by TSO

Power flow in nodal systems



- Linearized power flow equations
- While representing the power flow and transmission capacity limits, the resulting market-clearing optimization problem is linear and convex. Therefore, the solution obtained is proven to be globally optimal.
- Accordingly, the following equation gives the power flow across the transmission line connecting bus n to bus m :

$$f_{n,m} = B_{n,m}(\theta_n - \theta_m)$$

Power flow from bus n to bus m (variable) → $f_{n,m}$

Susceptance of the line connecting buses n to m (given data) → $B_{n,m}$

Difference of voltage angles of buses n and m (variable) → $(\theta_n - \theta_m)$

- Additionally, the following inequalities impose **limits** on the power flow across the transmission line connecting bus n to bus m :

$$-F_{n,m} \leq f_{n,m} \leq F_{n,m}$$

Capacity of the line connecting buses n to m (given data) → $F_{n,m}$

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Market-clearing optimization: **Nodal**



Compact form:

$$\begin{aligned} \text{Maximize}_{p_g^G, p_d^D, \theta_n} \quad & SW = \sum_d U_d \underbrace{(p_d^D)}_{\text{Consumption of demand } d} - \sum_g C_g \underbrace{(p_g^G)}_{\text{Production of producer } g} \\ \text{subject to:} \quad & 0 \leq p_d^D \leq \overline{P}_d^D \quad \forall d \\ & 0 \leq p_g^G \leq \overline{P}_g^G \quad \forall g \\ & \sum_{d \in \Psi_n} p_d^D + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^G = 0 \quad : \lambda_n \quad \forall n \\ & -F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m \\ & \theta_{ref} = 0 \end{aligned}$$

Set of primal variables: p_g^G ,

P_d^D , θ_n (Voltage angle of bus n)

SW: Social Welfare

Market-clearing optimization: **Nodal**



Compact form:

$$\text{Maximize}_{p_g^G, p_d^D, \theta_n} SW = \sum_d U_d p_d^D - \sum_g C_g p_g^G$$

subject to:

$$0 \leq p_d^D \leq \bar{P}_d^D \quad \forall d \longrightarrow \text{Consumption limits of demand } d$$

$$0 \leq p_g^G \leq \bar{P}_g^G \quad \forall g \longrightarrow \text{Production limits of producer } g$$

$$\sum_{d \in \Psi_n} p_d^D + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^G = 0 \quad : \lambda_n \quad \forall n$$

$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

More detail, check slides

Power Flow in Zonal systems

Power flow in zonal systems



- Neither AC nor linearized power flow equations are included.
- Power trade between each pair of connecting zones is restricted to NTCs (Net Transfer Capacities), as determined by TSOs (or RCCs).
- The day-ahead power trade between zones aa and bb is limited by:

$$-ATC_{a,b} \leq f_{a,b} \leq ATC_{a,b}$$

↑
Power transfer from
zone a to zone b
(variable)

↖
NTC between zones a and
 b (given parameter)

Examples of NTCs set by TSOs and the corresponding RCC:

Asymmetric NTCs: $-1200 \leq f_{a,b} \leq 2000$



Trade from zone a to zone b is limited to a maximum of 2000 MW, while trade in the opposite direction is limited to 1200 MW.

Nodal → Zonal

$$\text{Maximize}_{p_g^G, p_d^D, \theta_n} SW = \sum_d U_d p_d^D - \sum_g C_g p_g^G$$

subject to:

$$0 \leq p_d^D \leq \bar{P}_d^D \quad \forall d$$

$$0 \leq p_g^G \leq \bar{P}_g^G \quad \forall g$$

$$\sum_{d \in \Psi_n} p_d^D + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^G = 0 \quad : \lambda_n \quad \forall n$$

$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

We remove this set of constraints and instead incorporate the NTC-based constraints (next slide).

Check others in slides

Market-clearing optimization: **Zonal**

$$\text{Maximize}_{p_g^G, p_d^D, f_{a,b}} SW = \sum_d U_d p_d^D - \sum_g C_g p_g^G$$

subject to:

$$0 \leq p_d^D \leq \bar{P}_d^D \quad \forall d$$

$$0 \leq p_g^G \leq \bar{P}_g^G \quad \forall g$$

$$\sum_{d \in \Psi_a} p_d^D + \sum_{b \in \Omega_a} f_{a,b} - \sum_{g \in \Psi_a} p_g^G = 0 \quad : \lambda_a^{\text{Zonal}} \quad \forall a$$

$$-ATC_{a,b} \leq f_{a,b} \leq ATC_{a,b} \quad \forall a, \forall b \in \Omega_a$$

5. Flow-based market coupling in Europe

- It's an alternative to the NTC-based zonal model.
- The central western european (CWE) TSOs successfully implemented flow-based capacity calculation and allocation, with the go-live date on May 21st, 2015

Flow-based market coupling in Europe



Objective

- Improve the efficiency of NTC-based zonal market clearing by incorporating network constraints of critical domestic (intra-zonal) lines.
- Retain the zonal structure, ensuring a single price for each bidding zone.
- Maintain the market-clearing optimization problem as linear and convex.

This is achieved by incorporating **polyhedral constraints** on the zonal net injections within the market-clearing optimization model.

However, it remains far from the complexity of the nodal model.

6. Kahoot

- Q1: Which of the following statements about the day-ahead market in Nordpool is incorrect?
B: The market time unit is currently set to 30-minute intervals (1 hour in the time lecture deliver 2025)
- Q2: Which of the following statements about SDAC (single day-ahead coupling) is incorrect?
 - It takes around 5 hours to solve the market clearing optimisation problem (it took around 16 minutes)
- The system price is 102.32 Euros/MWh, What does this mean.
 - The unconstrained market-clearing price (if there were no grid limits)
- Which of the following statement is incorrect?

Which of the following statements is *incorrect*?

34

11

Answers

- ▲ There are **no** limits on power trade between DK1 and SE4.
- ◆ If there were no network limits, the price in SE2 would likely decrease.
- Demands in SE3 would benefit if there were no congestion between SE3 & SE4.
- If there were no network limits, the price in DK1 would likely decrease.

Incorrect: If there were not network limits, the price in SE2 would likely decrease.

- SE2 price is low 40.78 due to congestion, if there is no congestion, the generator can transfer power to other zones -> then the price in SE2 will be higher.

Q3:

The line capacity is 90 MW. What are the prices at Bus 1 (€/MWh) and Bus 2 (€/MWh), and what is the congestion rent (€)?

45

30, 35, 45

35, 35, 0

30, 35, 40

35, 50, 0

0 Answers

C: 35, 35, 0

It is not congested. Price difference is 0

Q4: The line capacity is 70 MW now, same question

The line capacity is 70 MW. What are the prices at Bus 1 (€/MWh) and Bus 2 (€/MWh), and what is the congestion rent (€)?

42

35, 35, 40

35, 35, 0

30, 35, 35

30, 35, 40

1 Answer

B: 30, 35, 35 = 7 x 5 = Euros

Q5: Which of the following statements is correct regarding grid-constrained market clearing under a uniform pricing scheme?

-Correct: The grid owner is compensated in the event of congestion

- Red is incorrect, it should be: Total payment by demand = Congestion rent + Total payment to producers
- Blue: It is budget balance.

Q6: A system has 3 nodes 3 lines, & 4 producers. How many nodal balancees must be enforced in the optimisation problem.
A system with:

- **3 nodes**
 - **3 lines**
 - **4 producers**
- requires **one nodal power balance equation per node**.

Number of nodal balances=3

Q7: A system has 3 nodes 3 lines, & 4 producers. How many LMPs (nodal prices) are there as dual variables?
1 LMP per node -> 3 LMPs, doesn't matter how many producers

Q8:

28

If an optimization problem is constrained by the AC power flow equations, ...

Skip

4
Answers

$$P_i = \sum_{k=1}^n |V_i||V_k|[g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)]$$
$$Q_i = \sum_{k=1}^n |V_i||V_k|[g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k)]$$

▲ it becomes non-linear but convex.

◆ it becomes non-convex.

● its solution is still guaranteed to be globally optimal.

■ no optimal solution, either local or global, can be obtained.

Blue is correct: It becomes non-convex.

Green is wrong: We can not obtain the optimal solution because we don't know it is either local or global

Q9: When obtaining the linearized DC power flow approximation from the AC equations, what approximation is not considered?

When obtaining the linearized DC power flow approximation from the AC equations, what approximation is *not* considered?

26

$$P_i = \sum_{k=1}^n |V_i||V_k|[g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^n |V_i||V_k|[g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k)]$$

20

▲ $b_{ik} = 0$

● $Q_i = 0$

◆ $|V_i| = |V_k| = 1$

■ $\sin(\theta_i - \theta_k) = \theta_i - \theta_k$

A is wrong , $b_{ik} = 0$

Q10: If the market-clearing optimisation problem (linear) is constrained by the linearized (DC) power flow equations,...
Its solution is guaranteed to be globally optimal.

Q11: What does the NTC-based model enforce for each bidding zone?

What does the NTC-based zonal model enforce for each bidding zone?

▲ 10

◆ 12

● 22 ✓

■ 44

Next

▲ Linearised power flow equations for the network inside the zone. ✕

● Power balance equation for the entire zone ✓

◆ Capacity of domestic lines inside the zone ✕

■ All ✕

Red is wrong, when we use NTC-based zonal, we ignore everything inside the zone.

- Capacity of all the domestic is not the same

Q12: Order the models in terms of their need for re-dispatch to restore grid feasibility, from lowest to highest magnitude:



Nodal AC, Nodal DC, Zonal flow-based coupling, Zonal NTC-based.

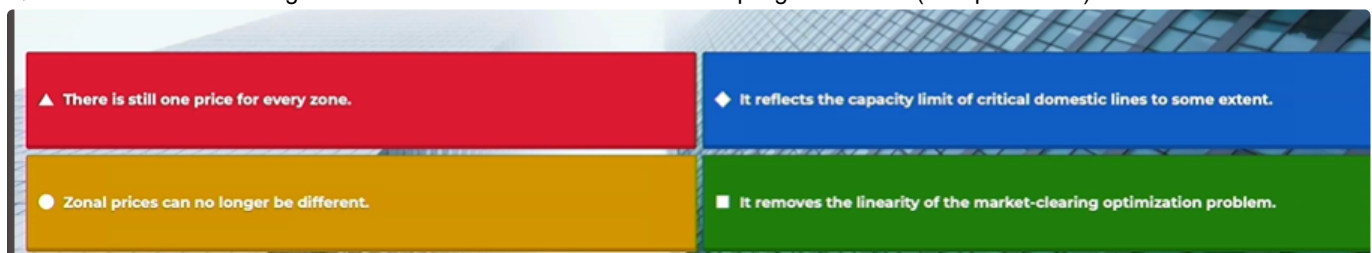
- Nodal AC: Respect power flow equation
- Nodal DC: Uses **linearized DC power flow**, Simple than AC, need a bit redispatch
- Zonal flow-based coupling: Don't see nodal structure at all, but try to add critical constraints using PTDF. Need more moderate redispatch
- **Zonal NTC-Based — highest redispatch**
- Only enforces **cross-border limits**.
- Completely ignores **internal grid constraints**.
- Market result often infeasible → TSOs must fix everything afterward.

Dispatch = the amount of power each generator is instructed to produce in a given hour (or 15-minute interval) to meet demand.

It is the **physical implementation** of the market result.

- **Day-ahead market clears** → gives *scheduled* production for each generator.
- **Dispatch** → the TSO ensures generators actually produce those quantities in real time.
- **Redispatch** → corrections made by the TSO if the market schedule violates grid constraints. (*adjustments* made afterward to fix congestion or maintain grid feasibility.)

Q13: Which of the following statements about flow-based market coupling are correct (multiple choice)



- There is still one price for every zone
- It reflects the capacity limit of critical domestic lines to some extent.