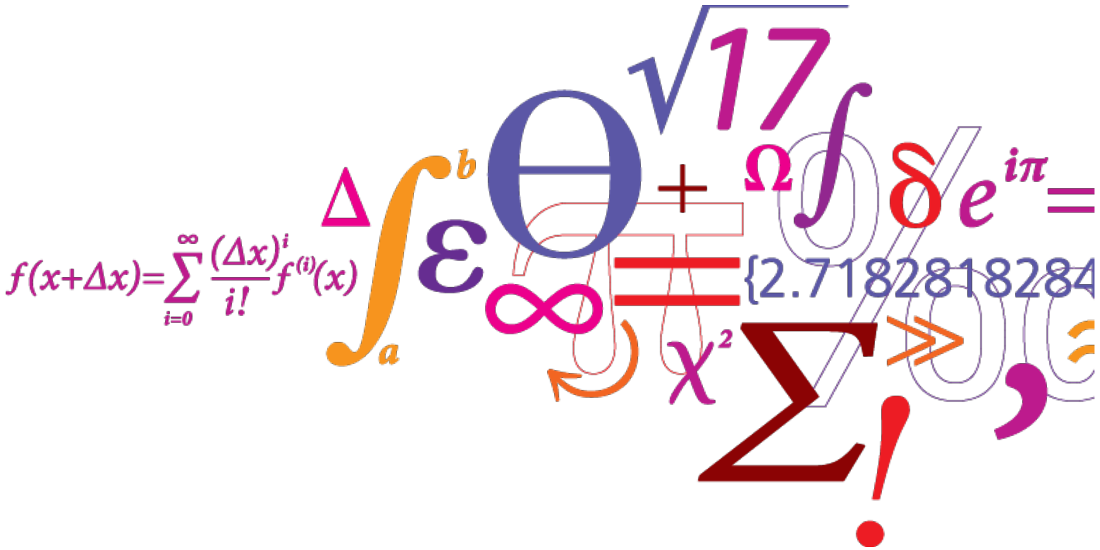
46755 – Renewables in Electricity Markets

Lecture 3: Day-ahead markets

Jalal Kazempour

February 17, 2025



Remember from the previous lecture ...

- Various market actors

  Power producers | Power demands | Market operator | System operators (TSO, DSO, ISO) | Market regulator | Traders | etc

- Various types of electricity market

  Capacity markets | Energy markets | Ancillary service (reserve) markets | Futures markets | Day-ahead markets | Intraday markets | Balancing markets

- European vs. U.S. electricity markets

  Nodal vs. zonal | ISO vs. TSO | Energy and reserve dispatch | Block orders

- Market-clearing as a linear optimization problem

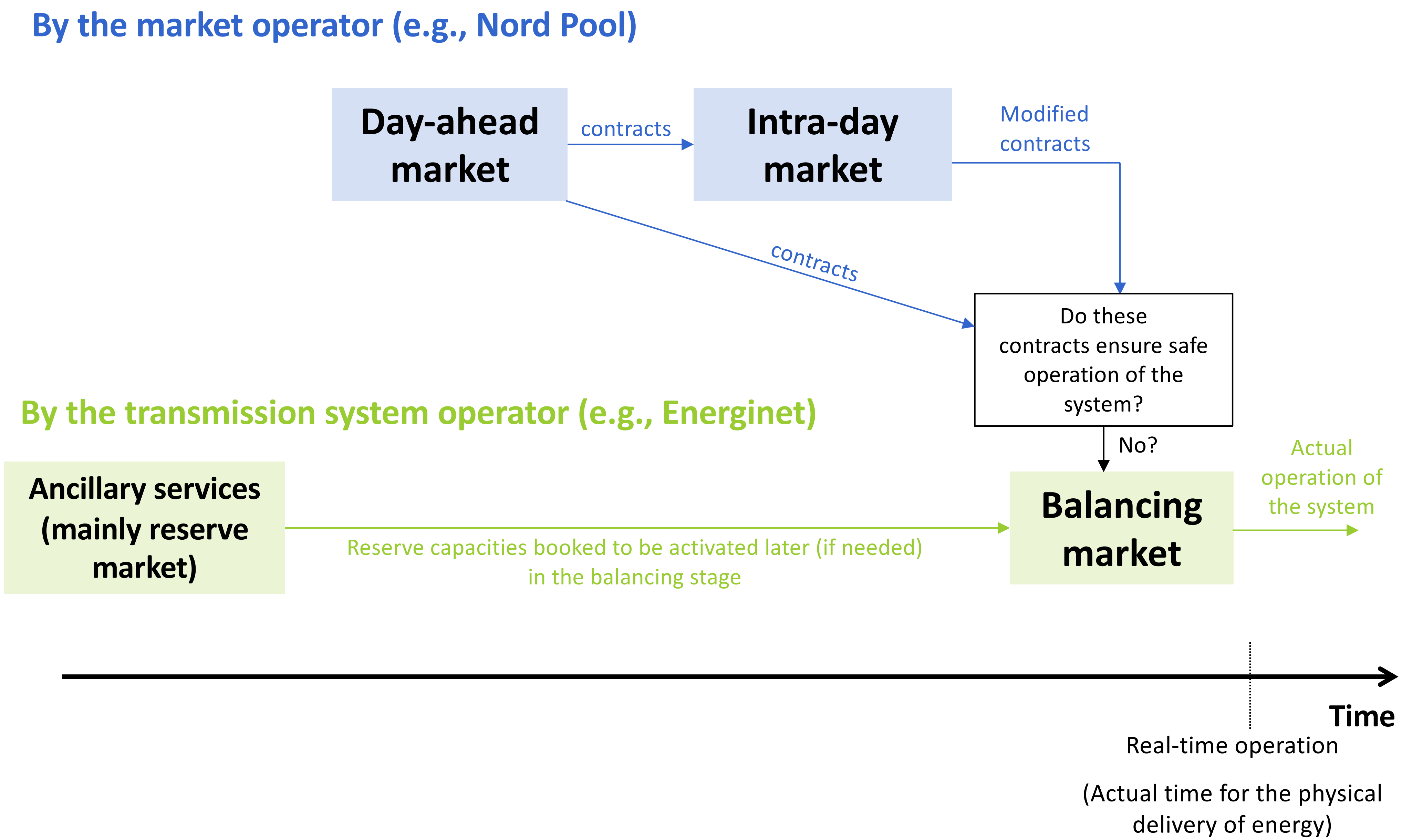
  Dual variables | Lagrangian function | KKT conditions| Price verification

Remember from the previous lecture ...

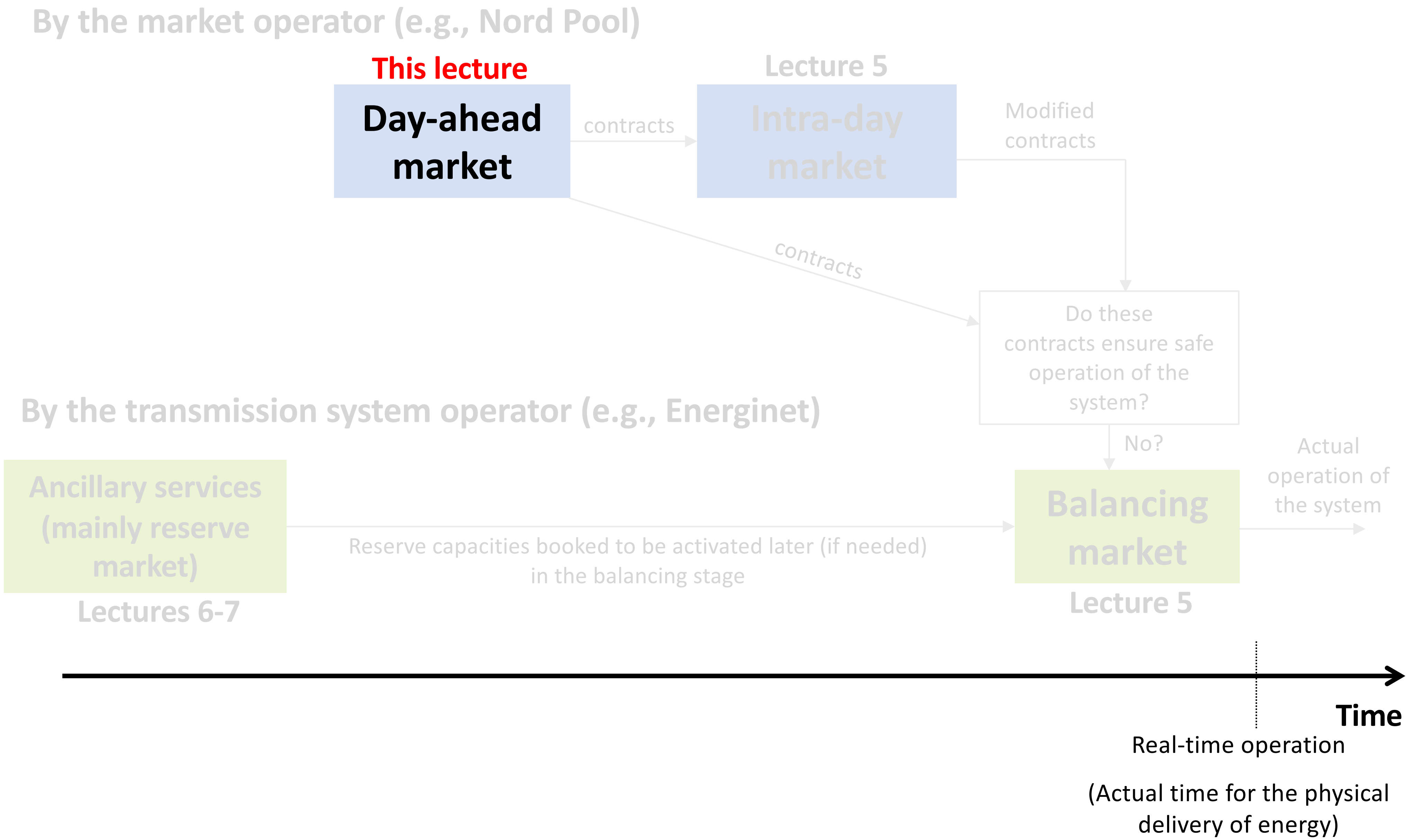
In Europe:
The transmission network is modeled in a simplified manner (using a **zonal** representation) within the day-ahead and intraday market-clearing process. There is no detailed network modeling within each bidding zone.

In the U.S.:
The transmission network is fully modeled using a **nodal** representation and a linearized power flow model in all market-clearing problems.

Remember from the previous lecture ...



Our focus in this lecture in on …



Learning objectives of this lecture

After this lecture, you are expected to:

- Explain the fundamentals of the day-ahead market.
- Describe the concept of day-ahead market coupling in Europe.
- Analyze the network effects in the market-clearing problem.
- Formulate the nodal and zonal day-ahead market-clearing optimization problems.
- Understand the concept of flow-based market coupling.

Outline

- Fundamentals of Day-Ahead Market Clearing
- Single Day-Ahead Coupling (SDAC) in Europe
- Network Effects on Market-Clearing Outcomes
- Power Flow Equations
- Market-Clearing Optimization Problem: Nodal vs. Zonal
- Flow-Based Market Coupling in Europe

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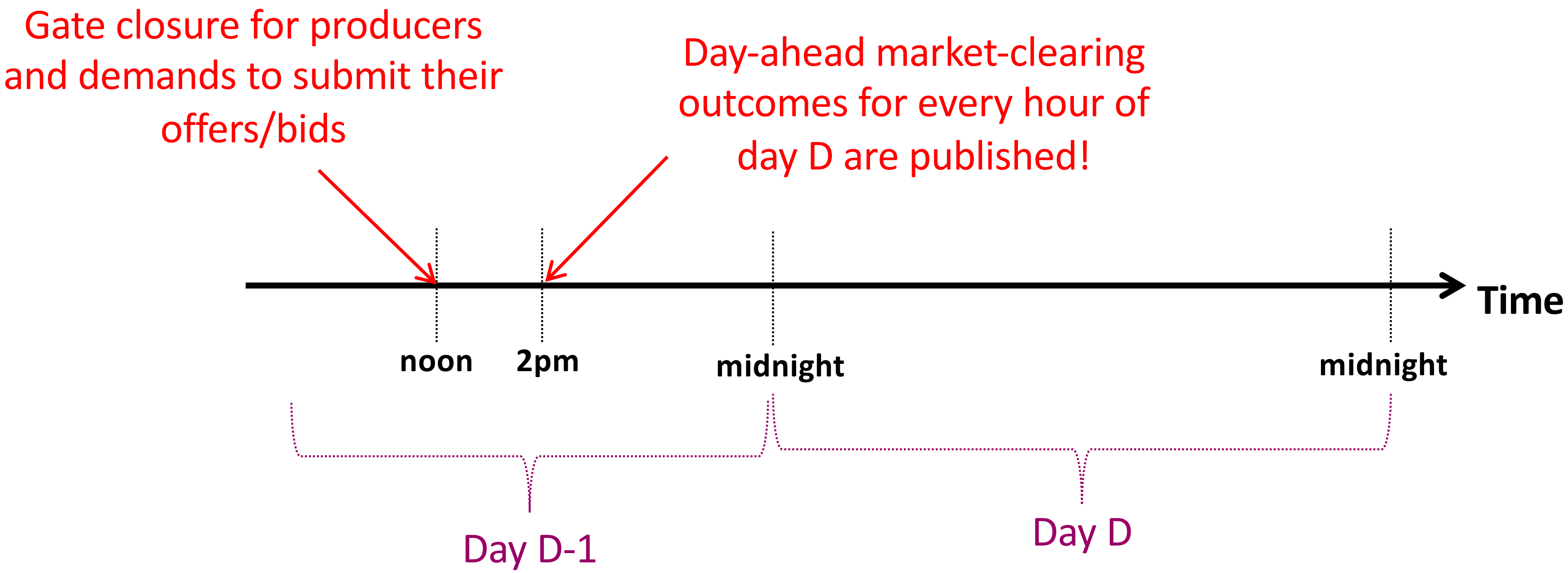
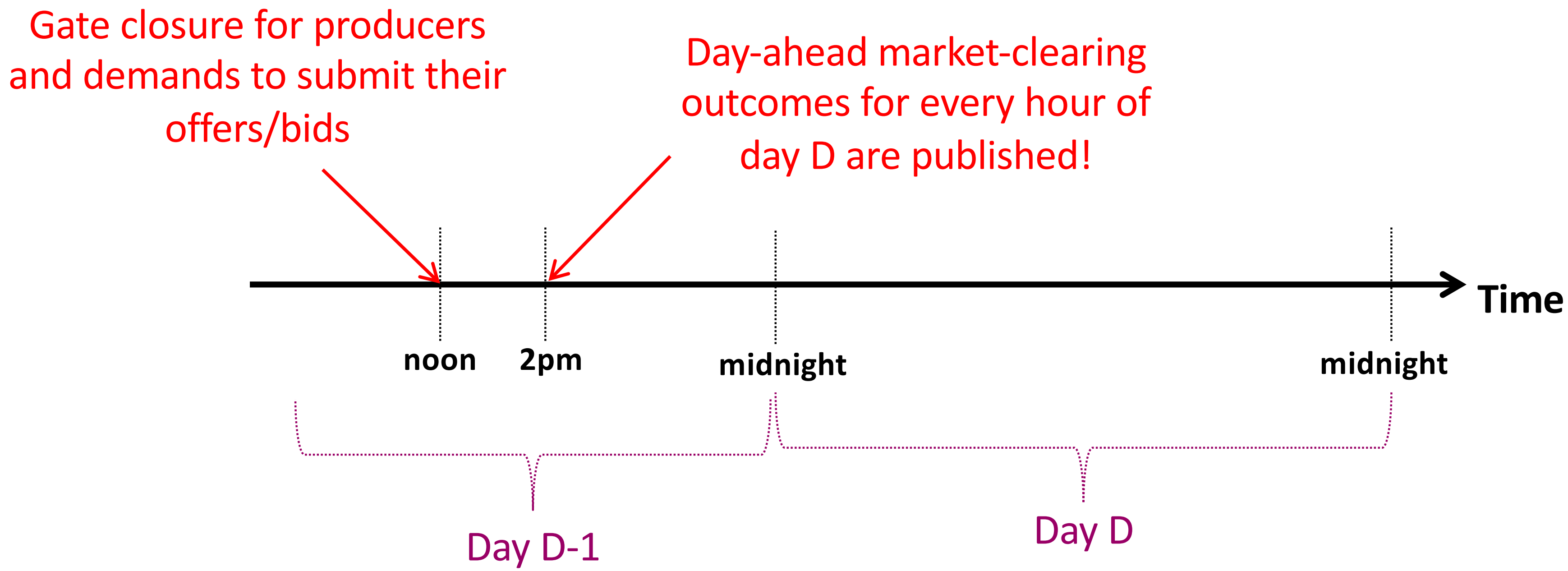
Day-ahead market in Nord Pool



On day D-1, the day-ahead market clearing determines production and consumption quantities, along with uniform market-clearing prices, for each hour of day D. These quantities and prices may vary from hour to hour but remain fixed within each hour.

Day-ahead market in Nord Pool



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- Time resolution for the day-ahead market, also referred to as market time unit (MTU):

  15-minute intervals starting June 11, 2025

Day-ahead market: Why?

Discussion:

Why is a day-ahead market necessary? Why not wait until real-time (or closer to real-time) to clear the electricity market instead?

Day-ahead market: Why?

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Why is a day-ahead market necessary? Why not wait until real-time (or closer to real-time) to clear the electricity market instead?

Operational reasons: Many conventional generation units, such as nuclear and coal plants, are slow and inflexible. They have various operational constraints and require advance scheduling to operate efficiently.

Day-ahead market: Why?

Discussion:

Why is a day-ahead market necessary? Why not wait until real-time (or closer to real-time) to clear the electricity market instead?

Operational reasons: Many conventional generation units, such as nuclear and coal plants, are slow and inflexible. They have various operational constraints and require advance scheduling to operate efficiently.

Economic reasons: Economic studies, e.g., [1], show that forward financial markets, any market cleared before real-time operation, enhance market competitiveness and reduce market power (strategic bidding). This prevents market participants from manipulating market-clearing outcomes for their own benefit.

[1] A. Blaise and V. Jean-Luc, "Cournot competition, forward markets and efficiency," *Journal of Economic Theory*, vol. 59, no. 1, pp. 1–16, Feb. 1993.

Day-ahead market: Why?

In future 100% renewable-based power systems with massive integration of distributed energy resources, day-ahead markets may not be necessary. Instead, long-term futures markets and near real-time operational markets might be sufficient.

- Alternatively, could “**electricity as a service**” be the solution?

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SDAC: Single Day-Ahead Coupling

Source: ENTSO-E [link]

- Launched in February 2014

SDAC: Single Day-Ahead Coupling

Source: ENTSO-E [link]

"The aim of Single Day-ahead Coupling (SDAC) is to create a single pan European cross zonal day-ahead electricity market. SDAC allocates scarce cross-border transmission capacity in the most efficient way by coupling wholesale electricity markets from different regions through a common algorithm, simultaneously taking into account cross-border transmission constraints, and thereby maximizing social welfare."

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An integrated day-ahead market increases the overall efficiency of trading by:

- *Promoting effective competition*
- *Increasing liquidity*
- *Enabling a more efficient utilization of the generation resources across Europe.*

SDAC: Single Day-Ahead Coupling

Source: ENTSO-E [link]

Involved parties

Transmission System Operators (TSOs):

50Hertz Transmission, ADMIE, Amprion, APG, AST, ČEPS, Creos, EirGrid, Elering, ELES, ELIA, Energinet, ESO, Fingrid, HOPS, Litgrid, MAVIR, PSE, REE, REN, RTE, SEPS, SONI, Statnett, Svenska Kraftnät, TenneT DE, TenneT NL, Terna, Transelectrica, and TransnetBW.

Nominated Electricity Market Operators (NEMOs):

BRM, BSP, CROPEX, SEMOpx (EirGrid and SONI), EPEX, EXAA, GME, HEnEx, HUPX, IBEX, Nord Pool, OMIE, OKTE, OPCOM, OTE, and TGE.

SDAC: Single Day-Ahead Coupling

Source: ENTSO-E [link]

Statistics on market

- *98,6% of EU consumption is coupled*
- *1.530 TWh/year coupled in one market solution*
- *200 M€ average daily value of matched trades*
- *17 minutes to solve a large and complex optimization problem*

How does SDAC work?

Source: ENTSO-E [link]

Pan-European Hybrid Electricity Market Integration Algorithm (EUPHEMIA)

is an algorithm developed to couple all different market rules, to calculate electricity prices across Europe, and to allocate cross-border capacity on a day-ahead basis.

For further reading, refer to the Nord Pool document [link], August 2024.

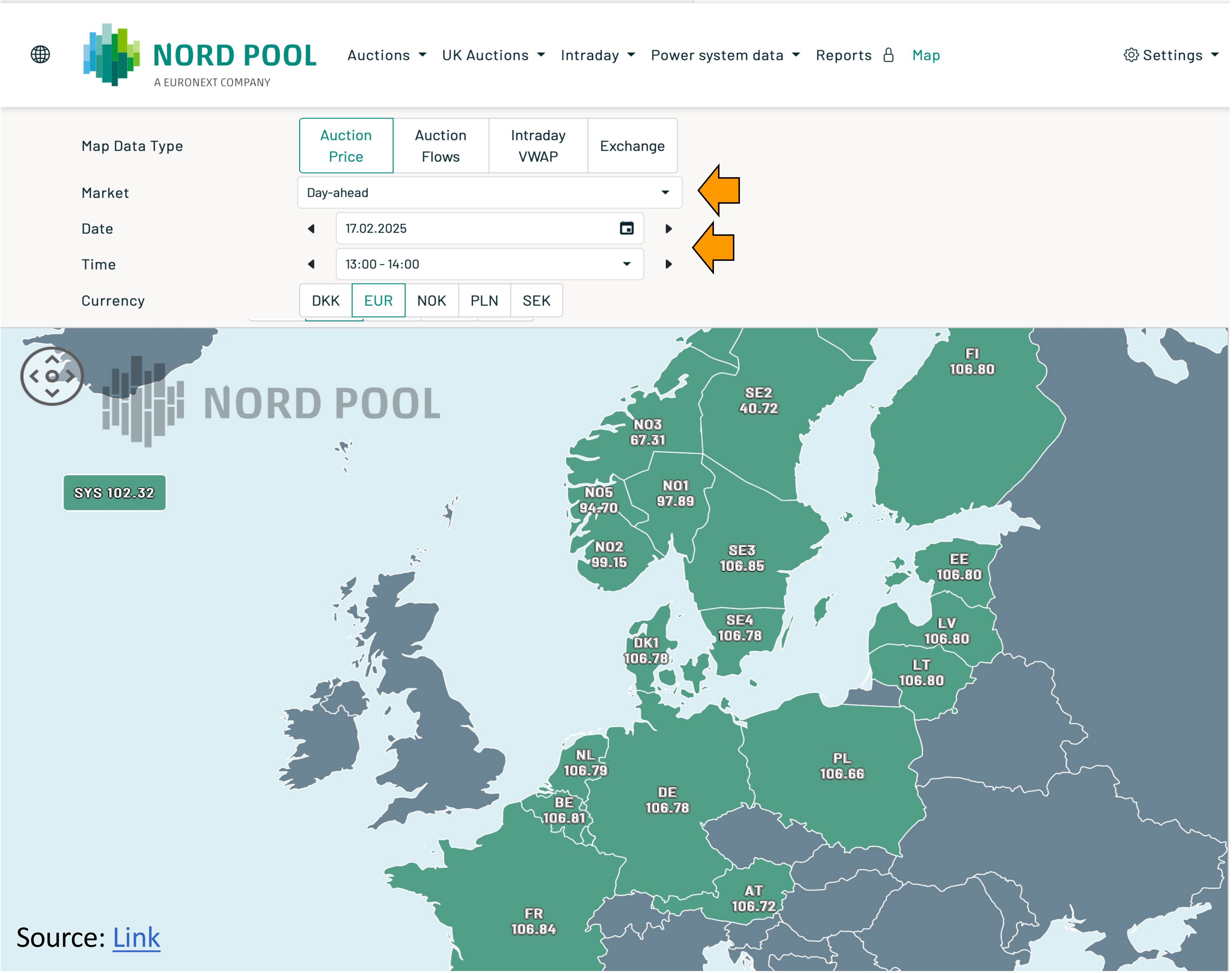
How does SDAC work?

Source: ENTSO-E [link]

“

- *Day-ahead market coupling requires processing input from all involved NEMOs and TSOs – essentially bids and offers and network capacities and constraints – matching them by operating one single algorithm, and lastly validating and sending outputs, such as matched trades, clearing prices, and scheduled exchanges, to NEMOs and TSOs.*
- *The SDAC makes use of a common price coupling algorithm, called PCR EUPHEMIA, to calculate electricity prices across Europe and to implicitly allocate auction-based cross-border capacity.*
- *Input data to PCR EUPHEMIA are the network capacities and constraints provided by the TSOs and the bids and offers provided by the NEMOs.*
- *PCR EUPHEMIA matches energy demand and supply for a full 24 hour-period.*
- *This process maximizes social welfare and takes into account price limits of orders and network constraints. The algorithm is designed to regard a large variety of orders and network features as well as local market rules.*
- *Output data are clearing prices, matched trades, scheduled exchanges, and the net position of bidding areas.*
- *Since its launch PCR EUPHEMIA has been continuously developed further.*

”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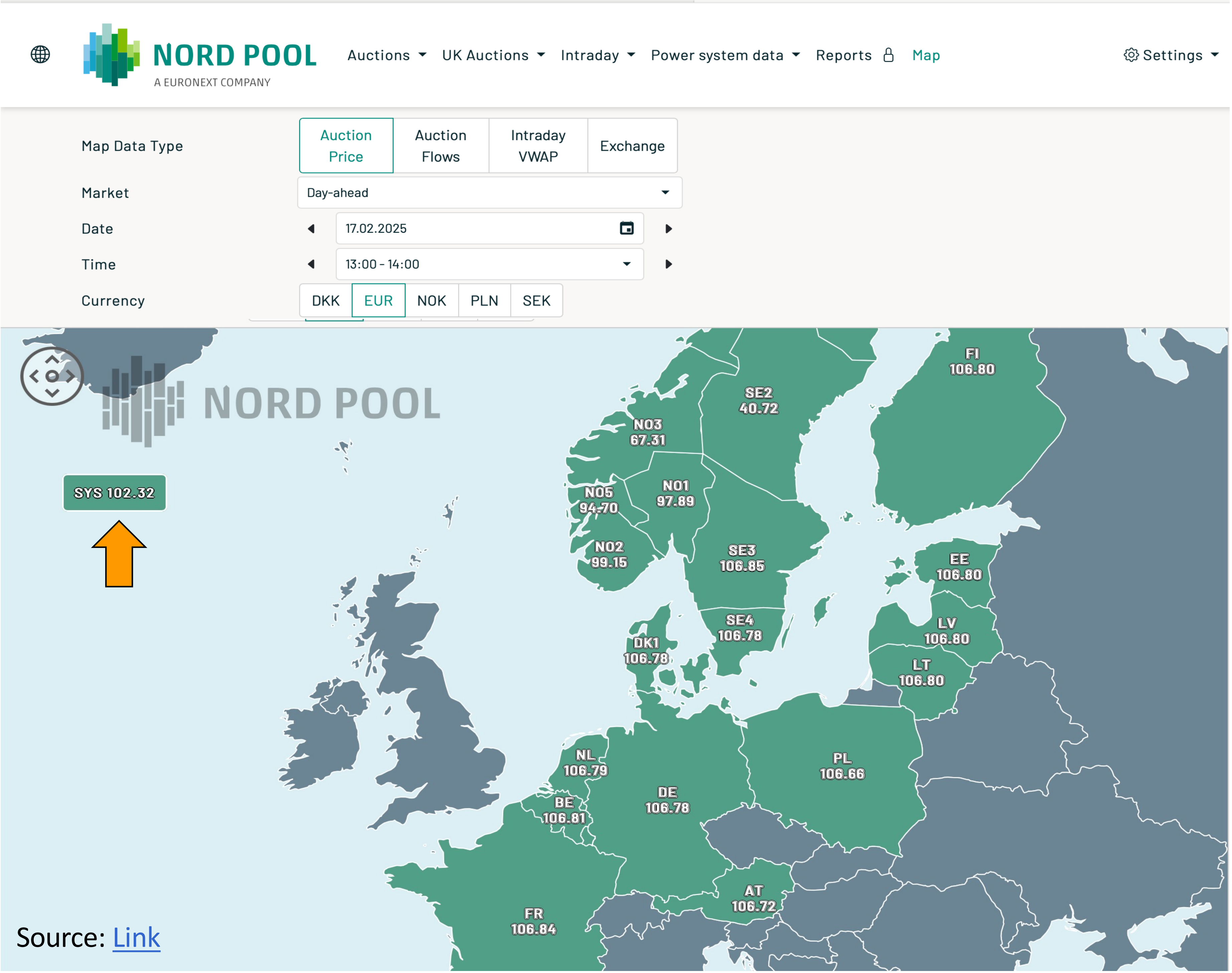
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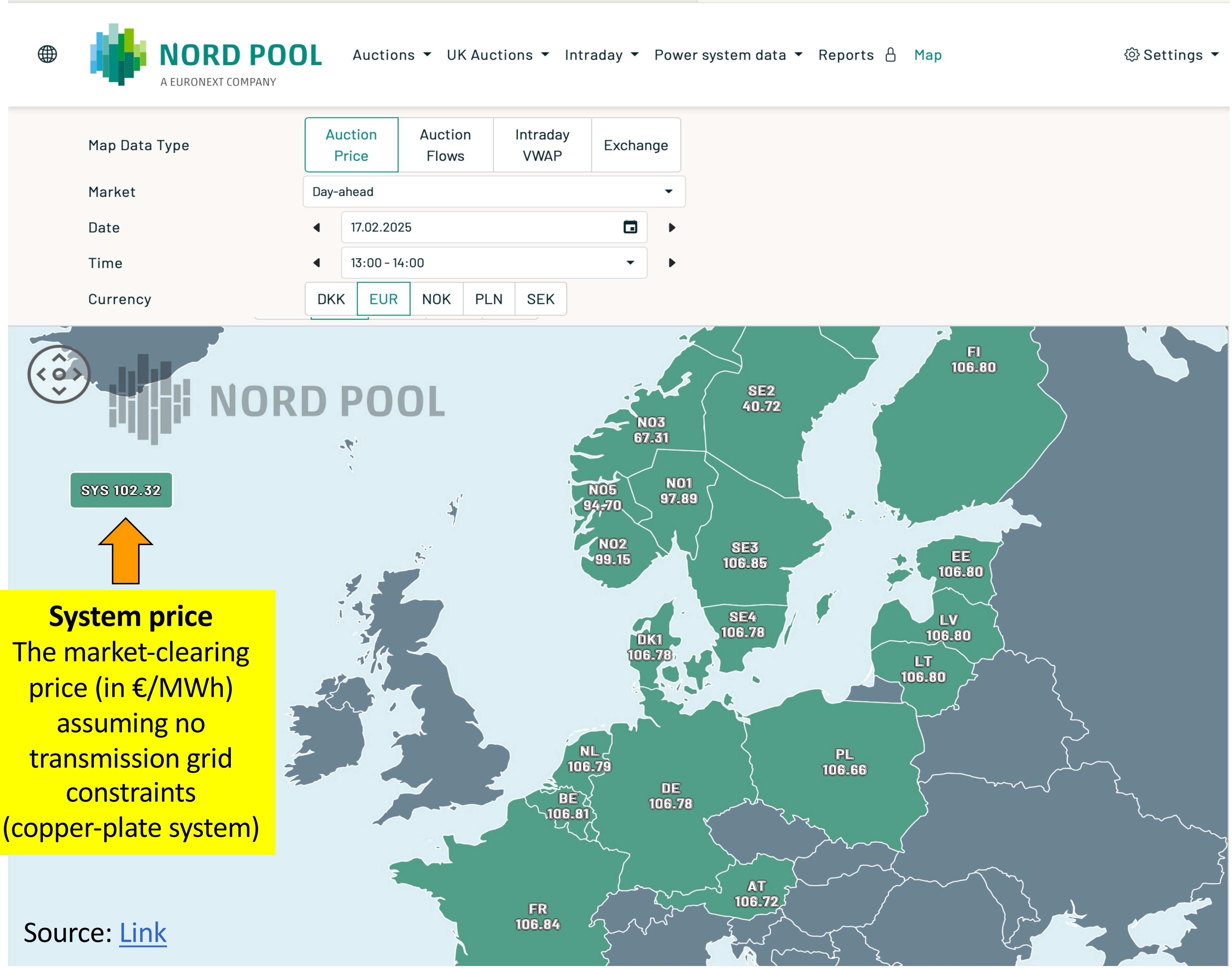
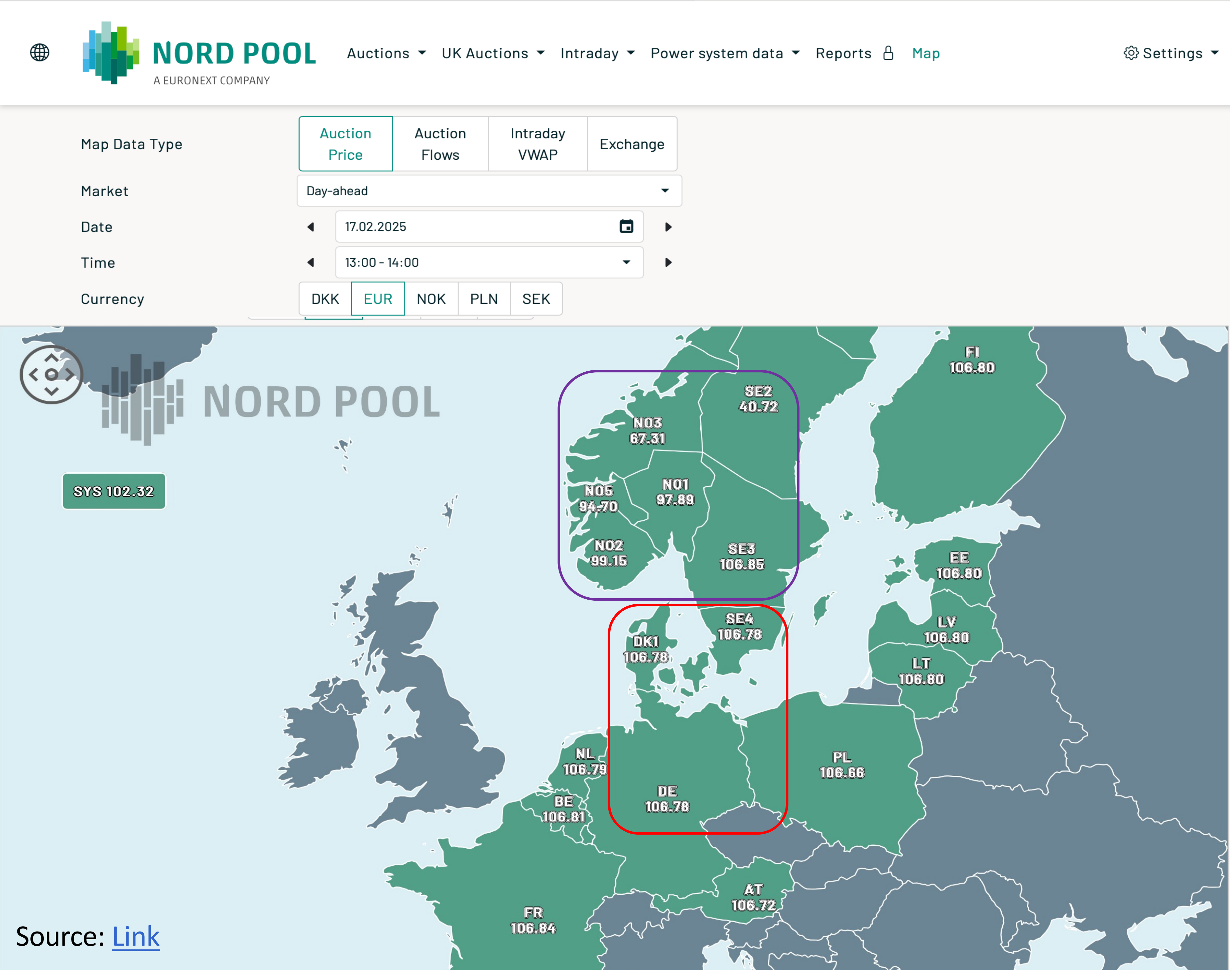


System price
The market-clearing price (in €/MWh) assuming no transmission grid constraints (copper-plate system)

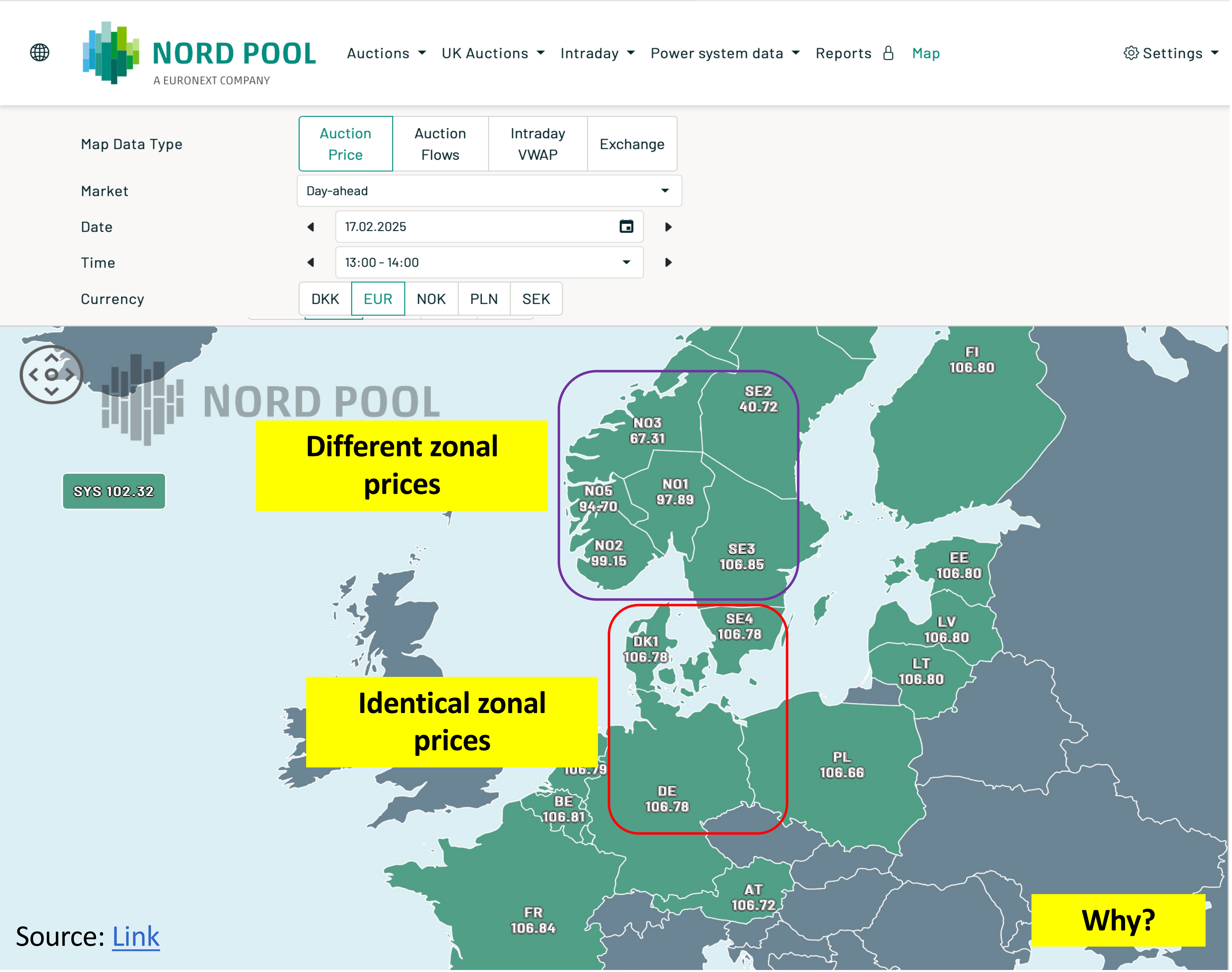
Source: Link



Source: Link

DTU Wind, Technical University of Denmark

Jalal Kazempour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An illustrative example

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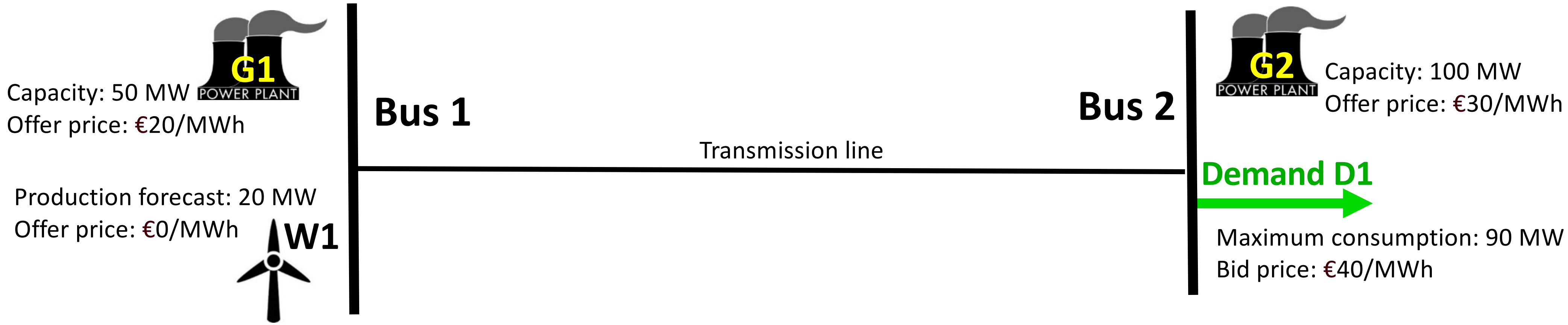

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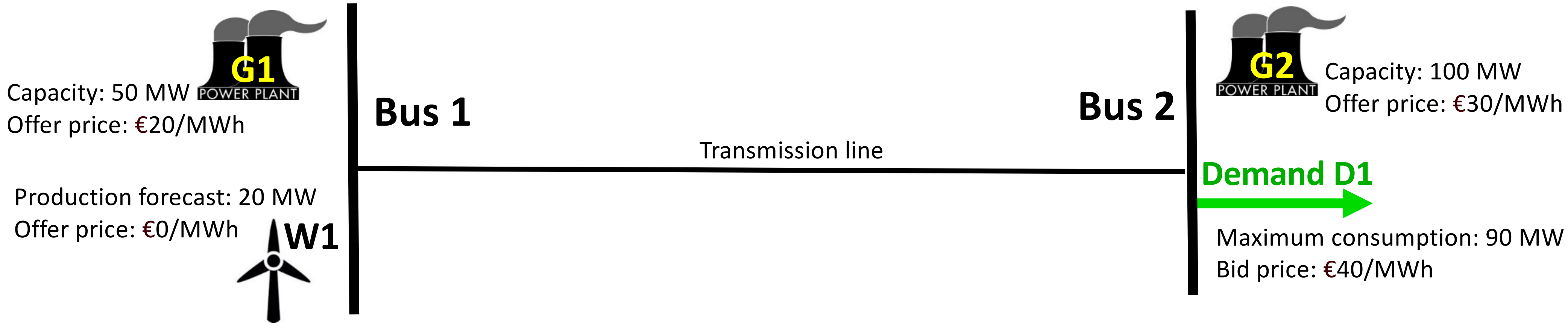

Note: We use the terms "**bus**" and "**node**", interchangeably.

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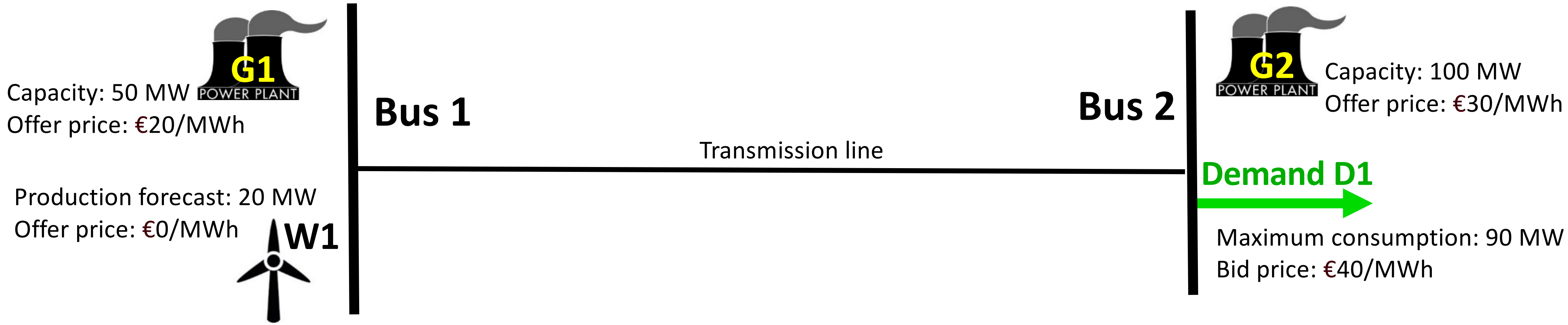

For the case where the transmission line capacity is 80 MW, clear the market:

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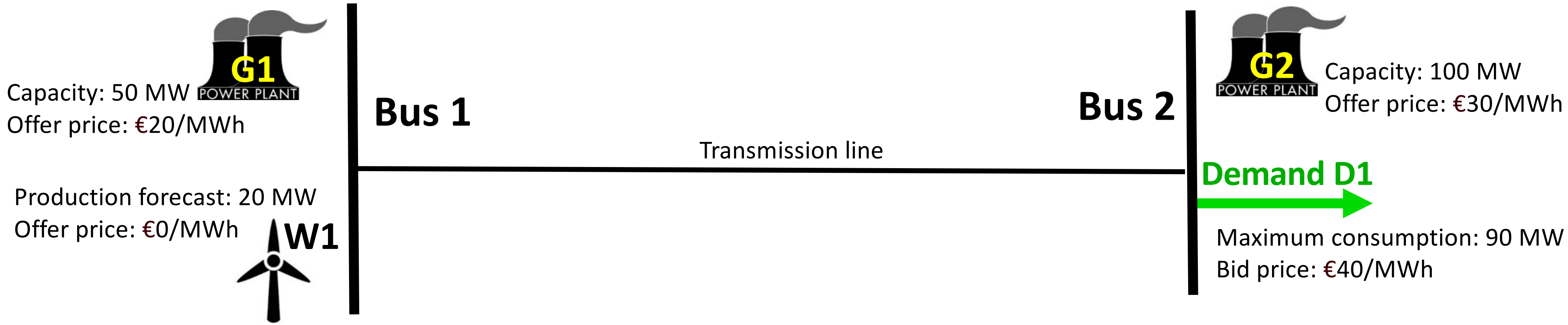

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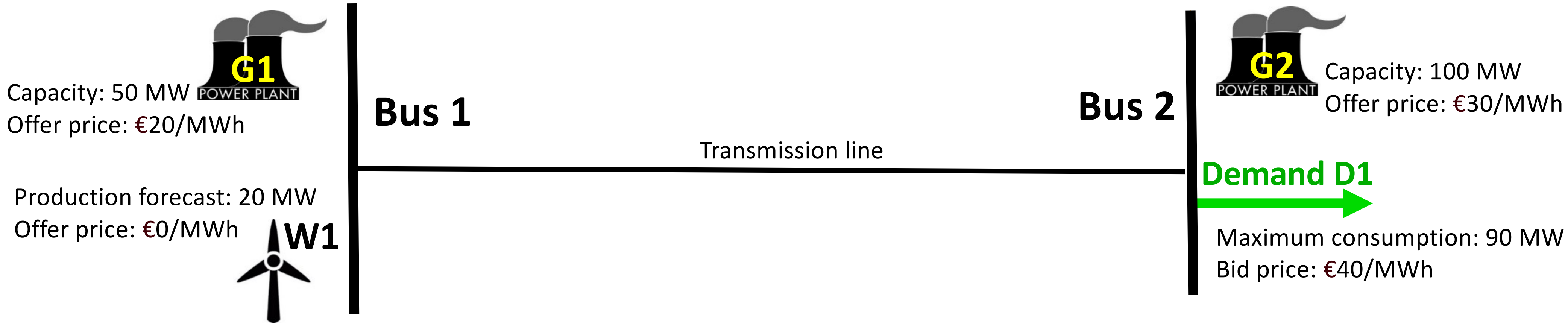

For the case where the transmission line capacity is 80 MW, clear the market:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **50**
- Production level (MW) of G2: **20**

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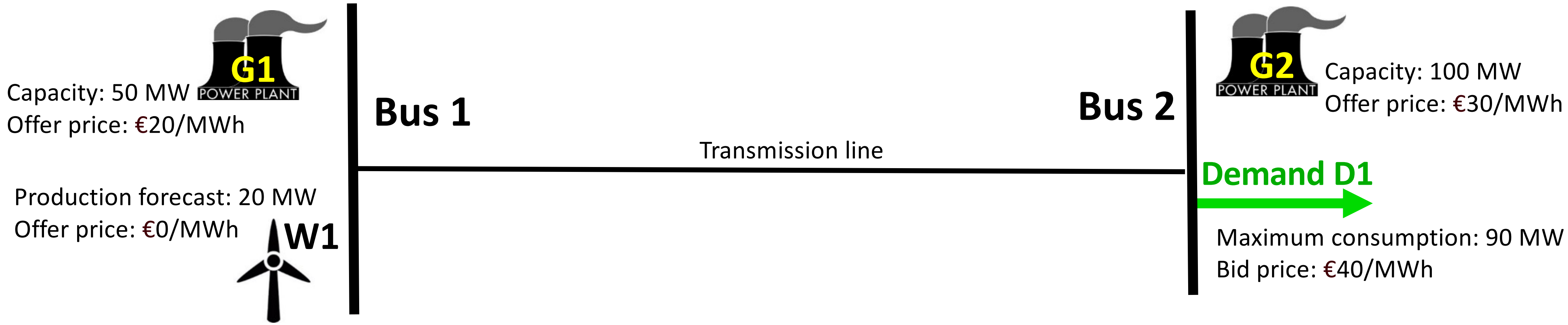

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- Power flow from Bus 1 to Bus 2 (MW): **?**

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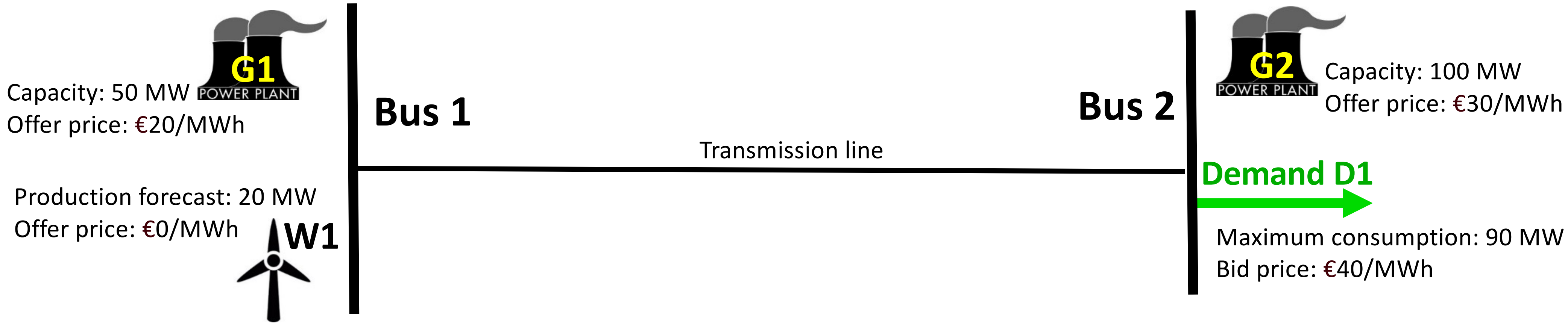

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- Production level (MW) of W1: **20**
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- Power flow from Bus 1 to Bus 2 (MW): **70**

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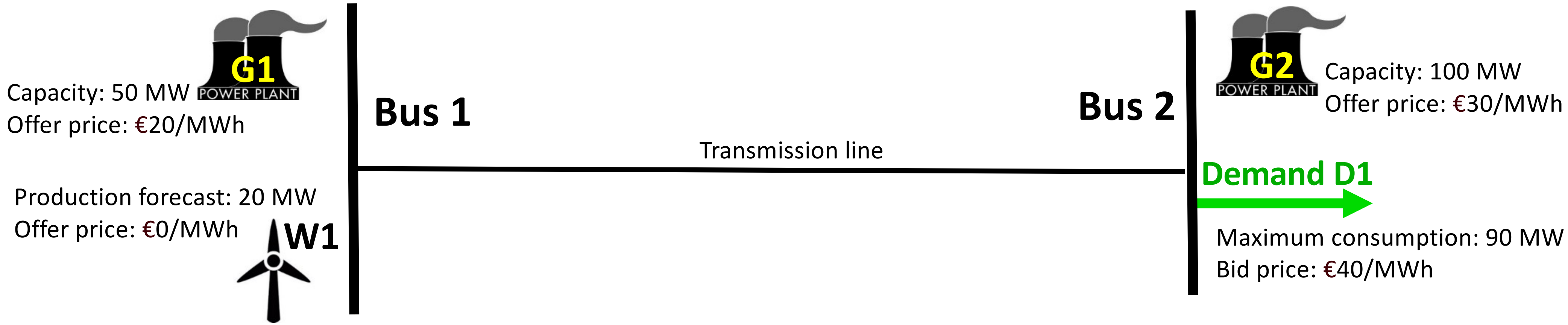

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- Market-clearing price (€/MWh) at Bus 1: **?**
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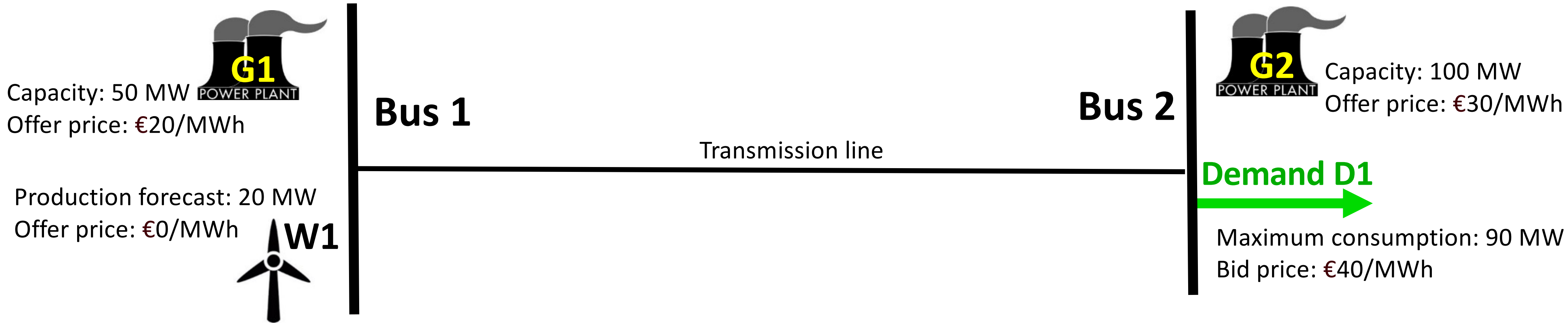

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- Production level (MW) of W1: **20**
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- Power flow from Bus 1 to Bus 2 (MW): **70**

- Market-clearing price (€/MWh) at Bus 1: **30**
- Market-clearing price (€/MWh) at Bus 2: **30**

Since any 1 MW change in demand (at either bus) results in a €30 change in the social welfare of the system.

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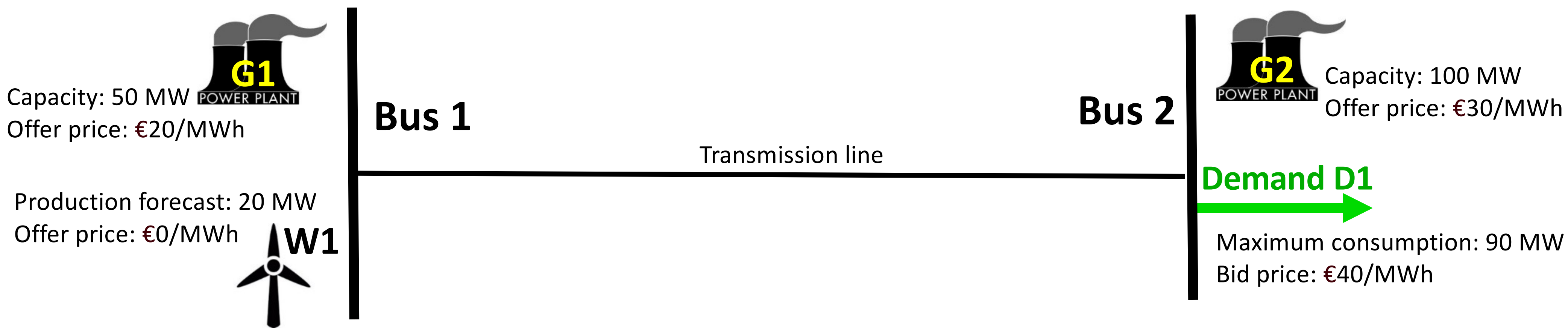

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- Power flow from Bus 1 to Bus 2 (MW): **70**

- Market-clearing price (€/MWh) at Bus 1: **30**
- Market-clearing price (€/MWh) at Bus 2: **30**

- Payment to W1 (€): ?
- Payment to G1 (€): ?
- Payment to G2 (€): ?
- Payment by D1 (€): ?

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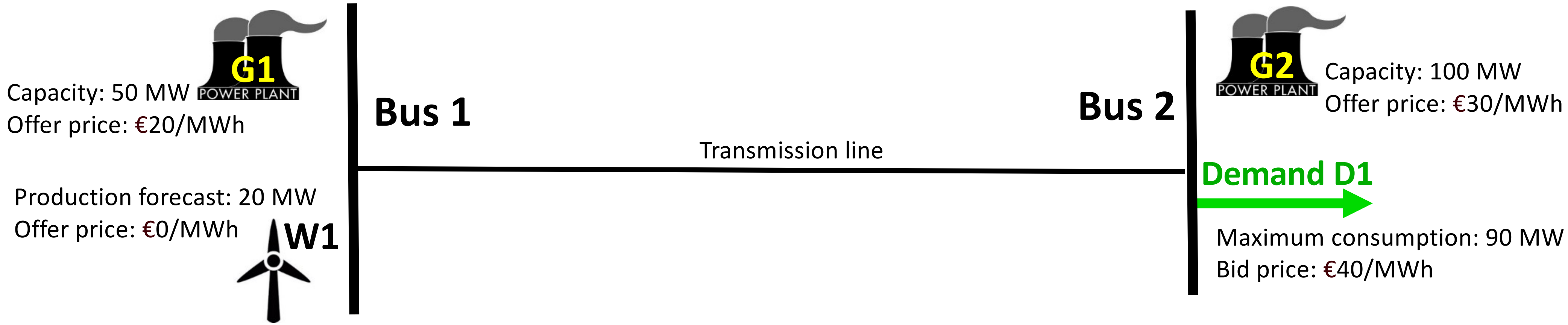

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- Power flow from Bus 1 to Bus 2 (MW): **70**

- Market-clearing price (€/MWh) at Bus 1: **30**
- Market-clearing price (€/MWh) at Bus 2: **30**

- Payment to W1 (€): 20×30=**600**
- Payment to G1 (€): 50×30=**1500**
- Payment to G2 (€): 20×30=**600**
- Payment by D1 (€): 90×30=**2700**

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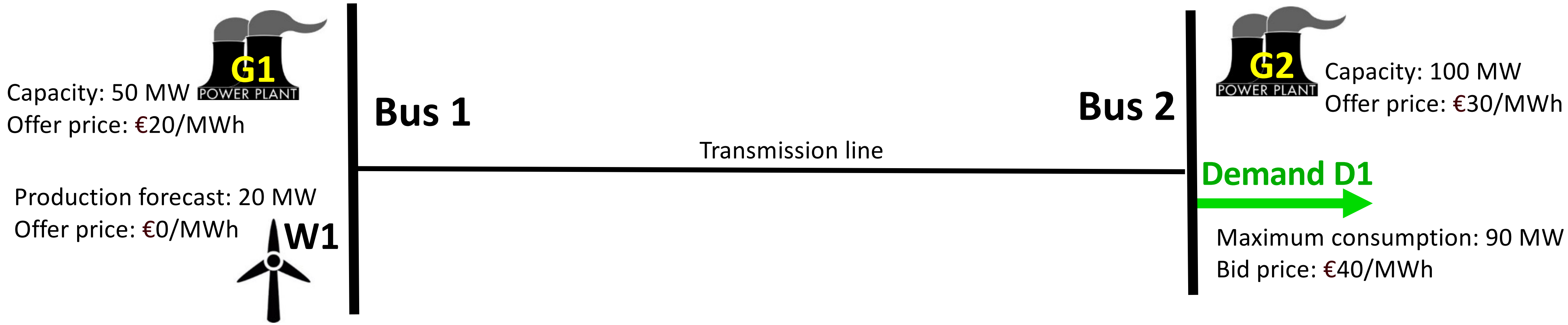

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- Payment to W1 (€): 20 × 30=**600**
- Payment to G1 (€): 50 × 30=**1500**
- Payment to G2 (€): 20 × 30=**600**
- Payment by D1 (€): 90 × 30=**2700**

Budget Balance:
Total payment to W1+G1+G2
= Payment by D1

An illustrative example

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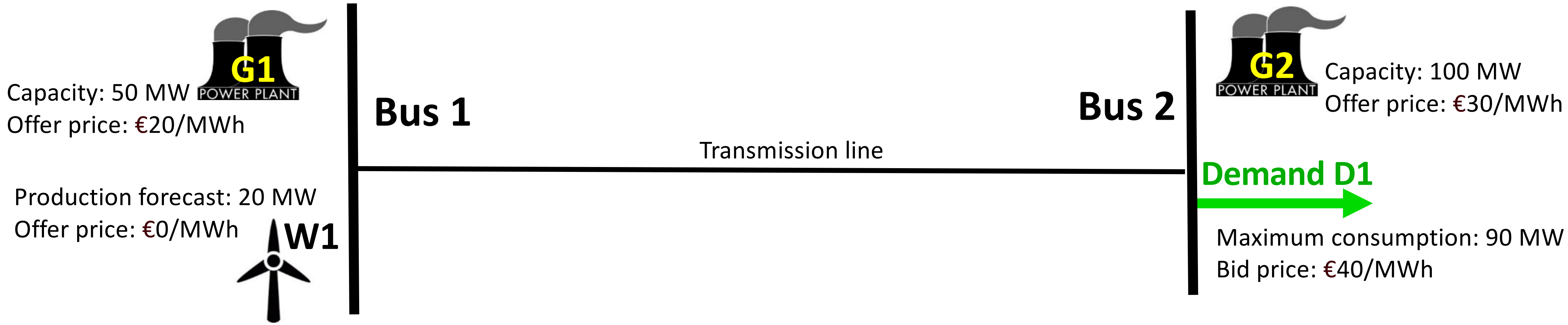

Now, we reduce the transmission line capacity to **60 MW**:

An illustrative example

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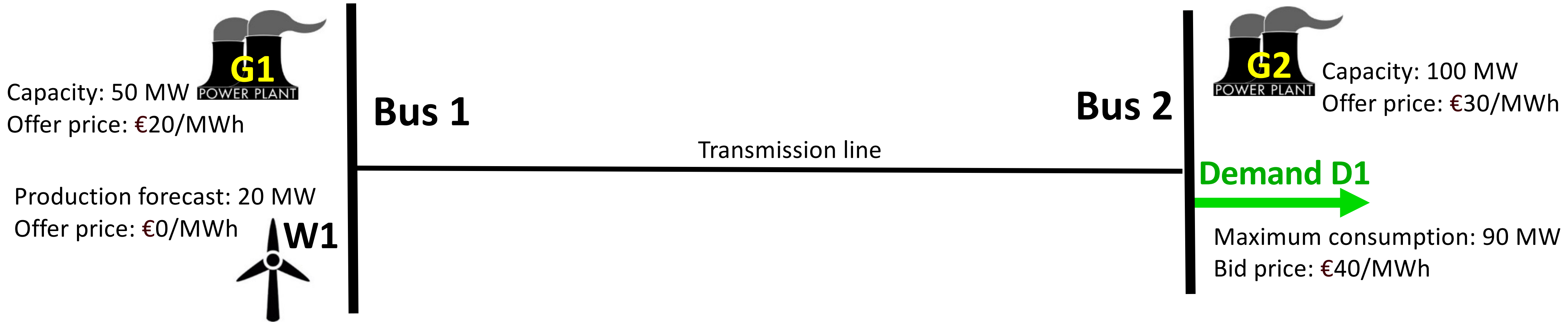

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- Power flow from Bus 1 to Bus 2 (MW): ?

- Market-clearing price (€/MWh) at Bus 1: ?
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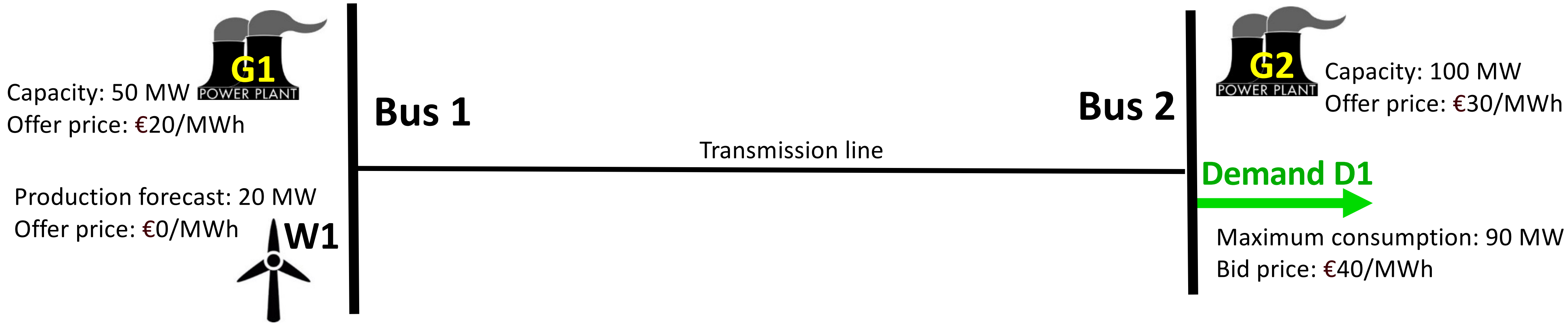


Now, we reduce the transmission line capacity to 60 MW:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **40**
- Production level (MW) of G2: **30**

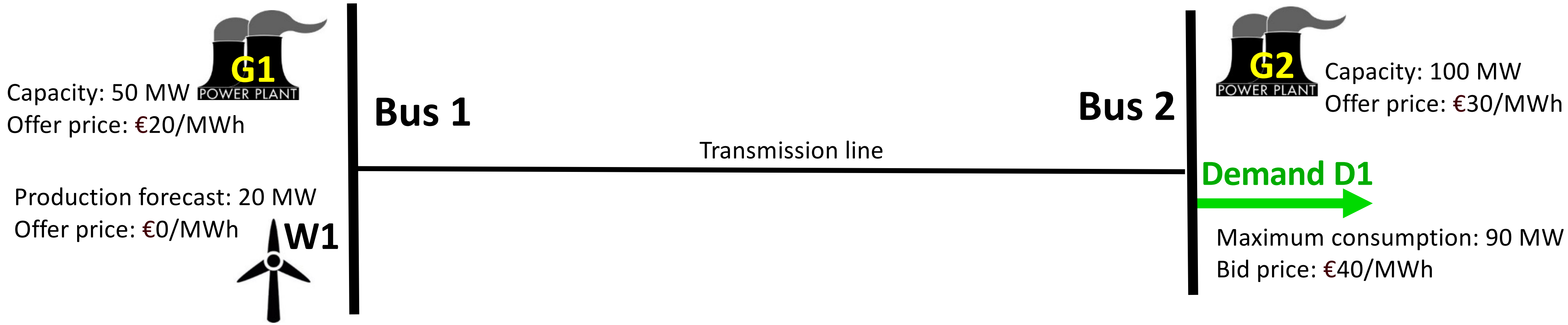
- Power flow from Bus 1 to Bus 2 (MW): **60 (congested)**

- Market-clearing price (€/MWh) at Bus 1: **20**
- Market-clearing price (€/MWh) at Bus 2: **30**

Why different prices?

An illustrative example

Let us consider this two-bus power network with three producers (W1, G1, and G2) and a single demand (D1):



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- Market-clearing price (€/MWh) at Bus 1: **20**
- Market-clearing price (€/MWh) at Bus 2: **30**

Since any 1 MW change in demand at bus 1 causes a €20 change in the social welfare of the system, while the same change at bus 2 results in a €30 change.

An illustrative example

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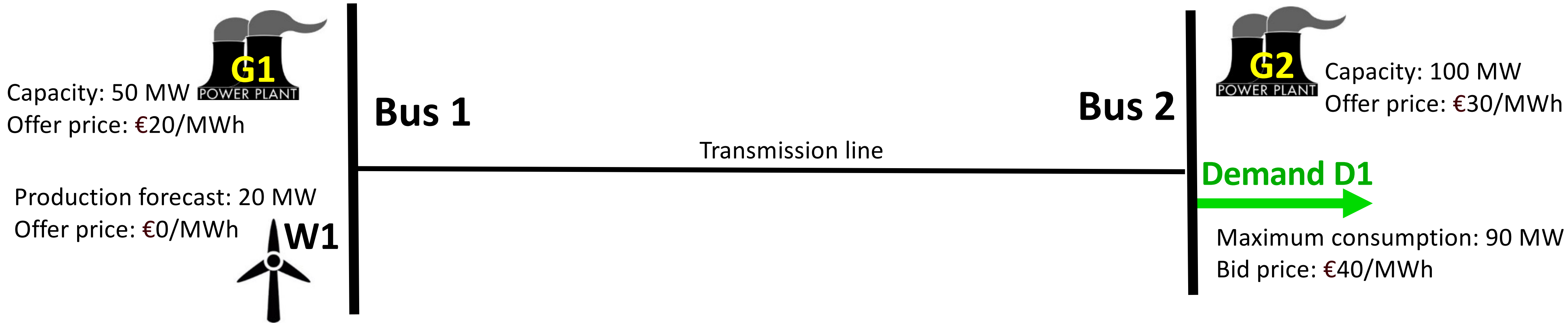

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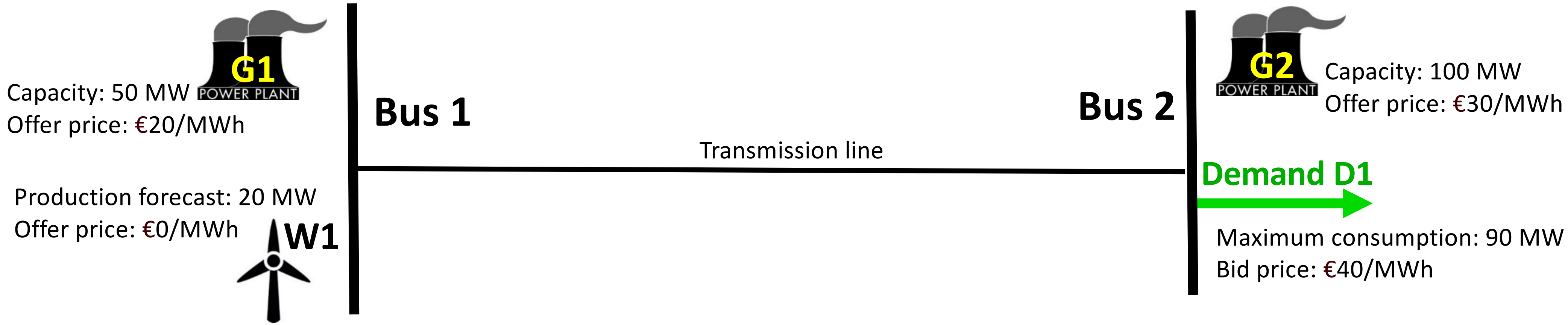
- Market-clearing price (€/MWh) at Bus 1: **20**
- Market-clearing price (€/MWh) at Bus 2: **30**

- Payment to W1 (€): ?
- Payment to G1 (€): ?
- Payment to G2 (€): ?
- Payment by D1 (€): ?

An illustrative example

Let us consider this two-bus power network with three producers (W1, G1, and G2) and a single demand (D1):



Now, we reduce the transmission line capacity to 60 MW:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **40**
- Production level (MW) of G2: **30**

- Power flow from Bus 1 to Bus 2 (MW): **60 (congested)**

- Market-clearing price (€/MWh) at Bus 1: **20**
- Market-clearing price (€/MWh) at Bus 2: **30**

- Payment to W1 (€): 20×20=**400**
- Payment to G1 (€): 40×20=**800**
- Payment to G2 (€): 30×30=**900**
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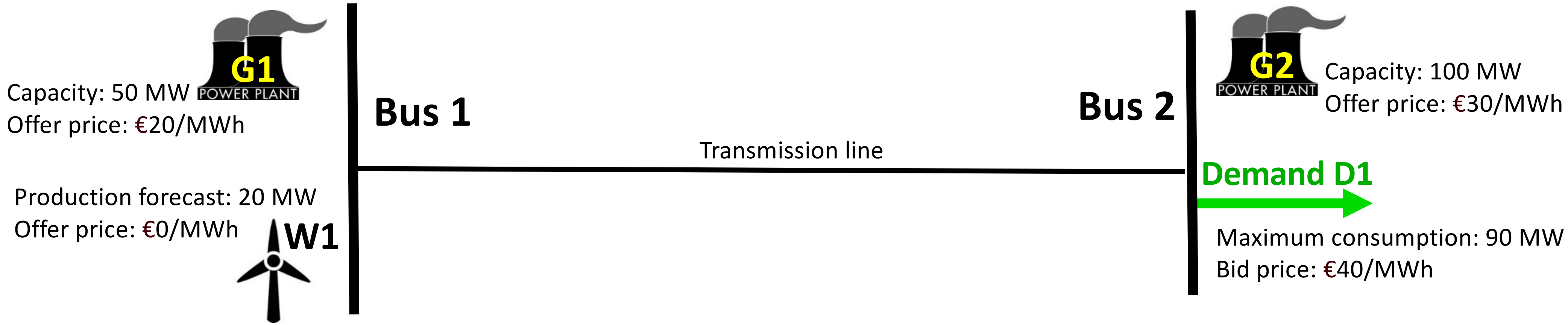

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- ○ Total payment to W1+G1+G2 = 400+800+900 = **€2100**
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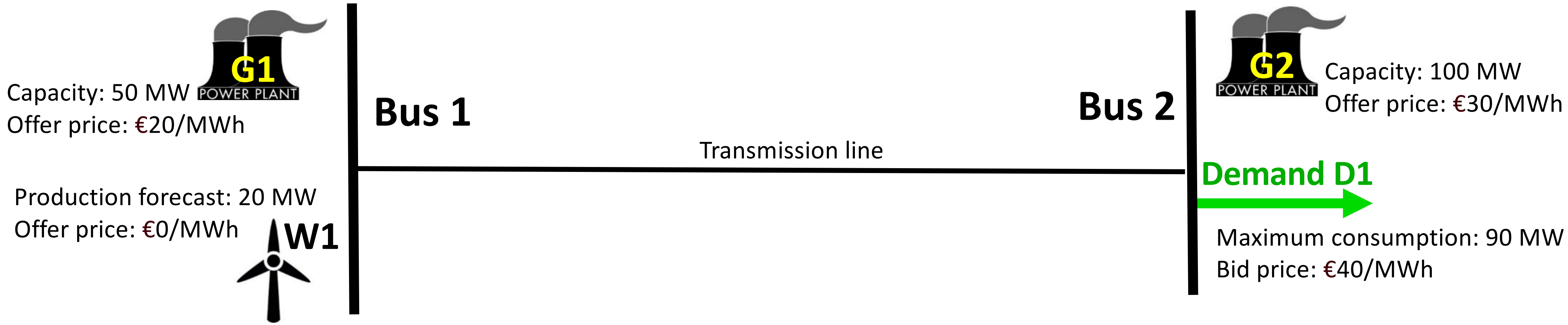

Now, we reduce the transmission line capacity to 60 MW:

Who is receiving the €600 surplus, and what does it represent?

- Payment to W1 (€): 20 × 20=**400**
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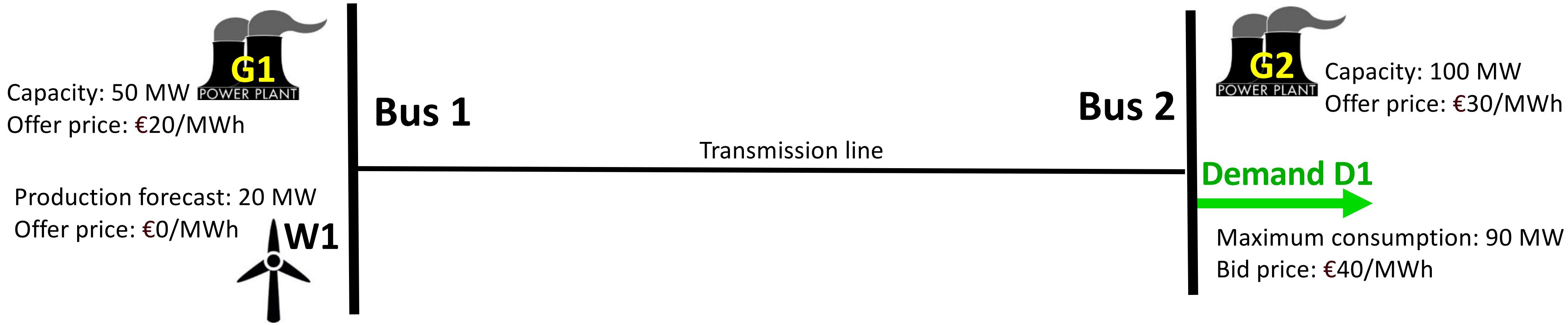

Now, we reduce the transmission line capacity to 60 MW:

Who is receiving the €600 surplus, and what does it represent?

It is called the "**congestion rent**", also referred to as "**congestion revenue**":

= Flow × [price at bus 2 − price at bus 1]

= 60 × [30 − 20] = 600.

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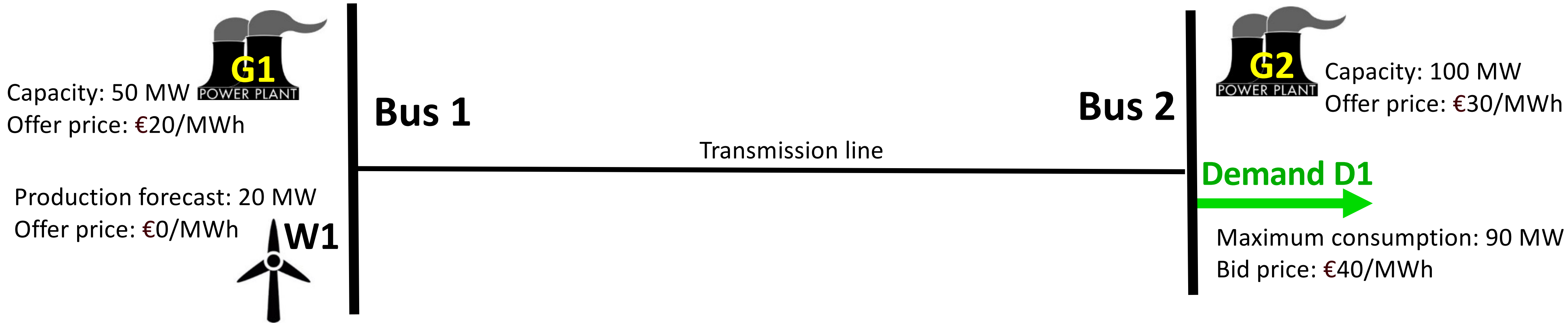

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= 60 × [30 − 20] = 600.

This surplus belongs to the grid owner (the TSO in this case). One can think of the grid owner as a "**spatial arbitrager**", buying 60 MW of power at bus 1 at the lower price of €20/MWh, transferring it to bus 2, and then selling it at the higher price of €30/MWh. This is similar to how storage acts as a **temporal arbitrager**.

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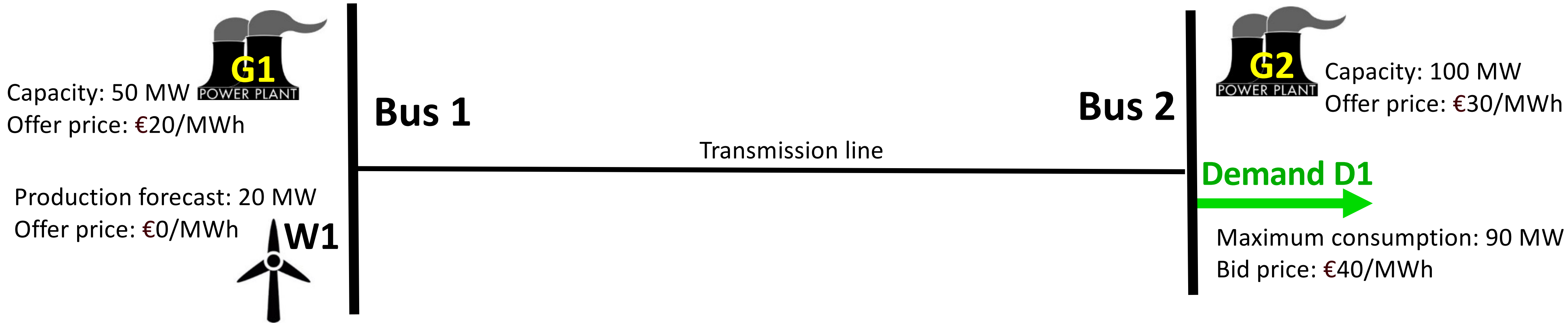

Congestion rent

- when the transmission limit is 80 MW (**uncongested case**):

  = 80 × [30 − 30] = €0.

- when the transmission limit is 60 MW (**congested case**):

  = 60 × [30 − 20] = €600.

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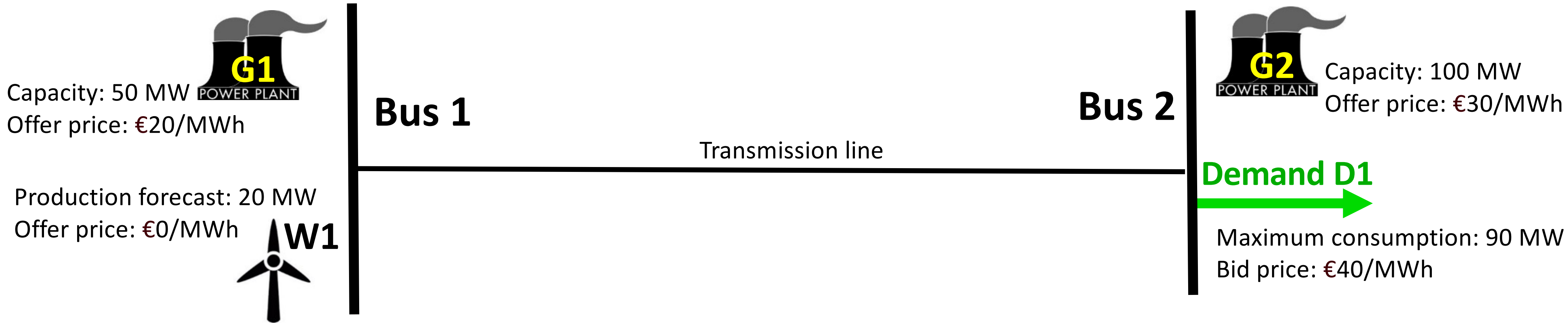

Congestion rent

- when the transmission limit is 80 MW (**uncongested case**):

  $= 80 \times [30 - 30] = €0.$

- when the transmission limit is 60 MW (**congested case**):

  $= 60 \times [30 - 20] = €600.$

Does this suggest that the TSO is a "for-profit" entity that earns revenue from congestion?

An illustrative example

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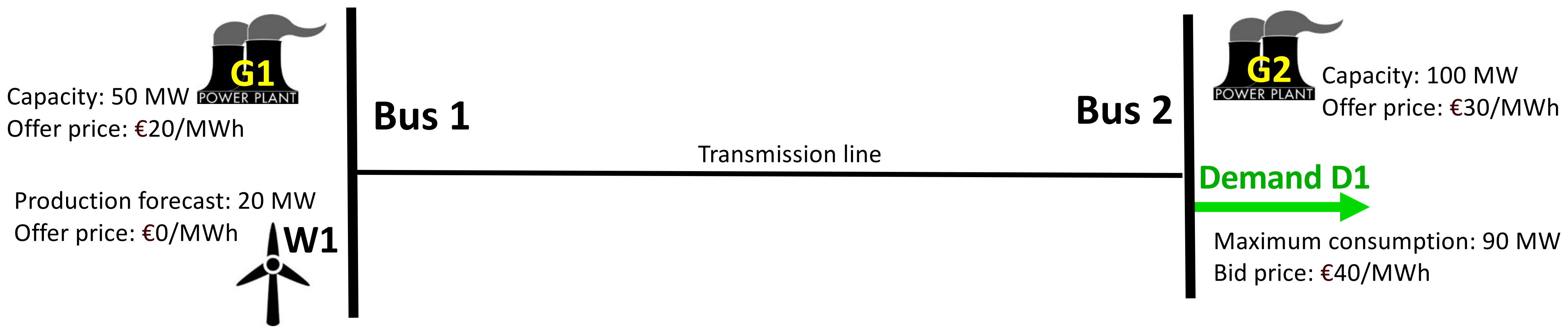

Budget Balance:
[Total payment to W1+G1+G2] + [Congestion rent] = Payment by D1

- Uniform pricing ensures budget balance, which is a desirable market property!

Market clearing as an optimization (with network)

$$\underset{p^{\mathrm{D1}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}},p^{1\to2}}{\text{Maximize}} \quad 40\ p^{\mathrm{D1}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

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$$-60 \leq p^{1\to2} \leq 60 \quad : \quad \underline{\mu}^{1\to2}, \overline{\mu}^{1\to2}$$

$$p^{1\to2} - p^{\mathrm{W1}} - p^{\mathrm{G1}} = 0 \quad : \quad \lambda^{\mathrm{Bus1}}$$

$$p^{\mathrm{D1}} - p^{1\to2} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda^{\mathrm{Bus2}}$$

Market clearing as an optimization (with network)

$$\underset{p^{D1},p^{W1},p^{G1},p^{G2},\boxed{p^{1\to 2}}}{\text{Maximize}} \quad 40\ p^{D1} - 0\ p^{W1} - 20\ p^{G1} - 30\ p^{G2}$$

New primal variable: Power flow (in MW) from bus 1 to bus 2

subject to:

$$0 \le p^{W1} \le 20 \quad : \quad \underline{\mu}^{W1}, \overline{\mu}^{W1}$$

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Power flow limits

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$$p^{\mathrm{D1}} - p^{1\to 2} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda^{\mathrm{Bus2}}$$

Question:
- Why is a lower bound (-60 MW) enforced?

Market clearing as an optimization (with network)

$$\underset{p^{D1},p^{W1},p^{G1},p^{G2},p^{1\to 2}}{\text{Maximize}} \quad 40\ p^{D1} - 0\ p^{W1} - 20\ p^{G1} - 30\ p^{G2}$$

subject to:

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$$\boxed{\begin{aligned} p^{1\to 2} - p^{W1} - p^{G1} &= 0 \quad : \quad \lambda^{\text{Bus1}} \\ p^{D1} - p^{1\to 2} - p^{G2} &= 0 \quad : \quad \lambda^{\text{Bus2}} \end{aligned}}$$

Power balance equalities (one per bus)

Market clearing as an optimization (with network)

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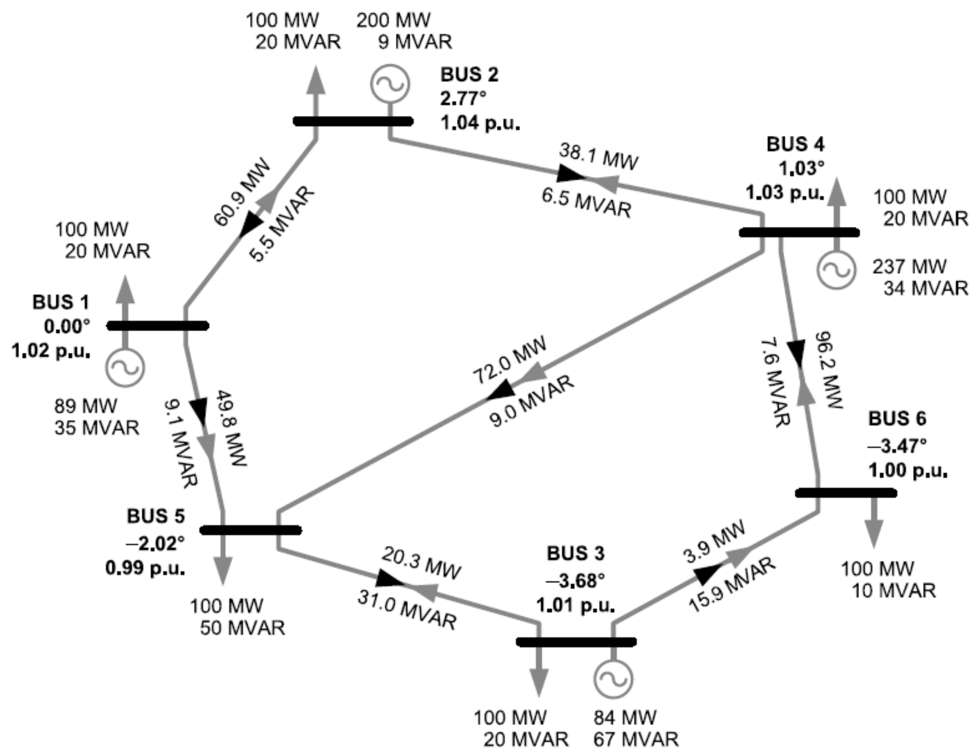
Nodal prices (one per bus)

Outline

- Fundamentals of Day-Ahead Market Clearing
- Single Day-Ahead Coupling (SDAC) in Europe
- Network Effects on Market-Clearing Outcomes
- Power Flow Equations
- Market-Clearing Optimization Problem: Nodal vs. Zonal
- Flow-Based Market Coupling in Europe

Power flow

Remember that the power flow in an electrical transmission network is governed by the laws of physics, such as Kirchhoff's and Ohm's laws.



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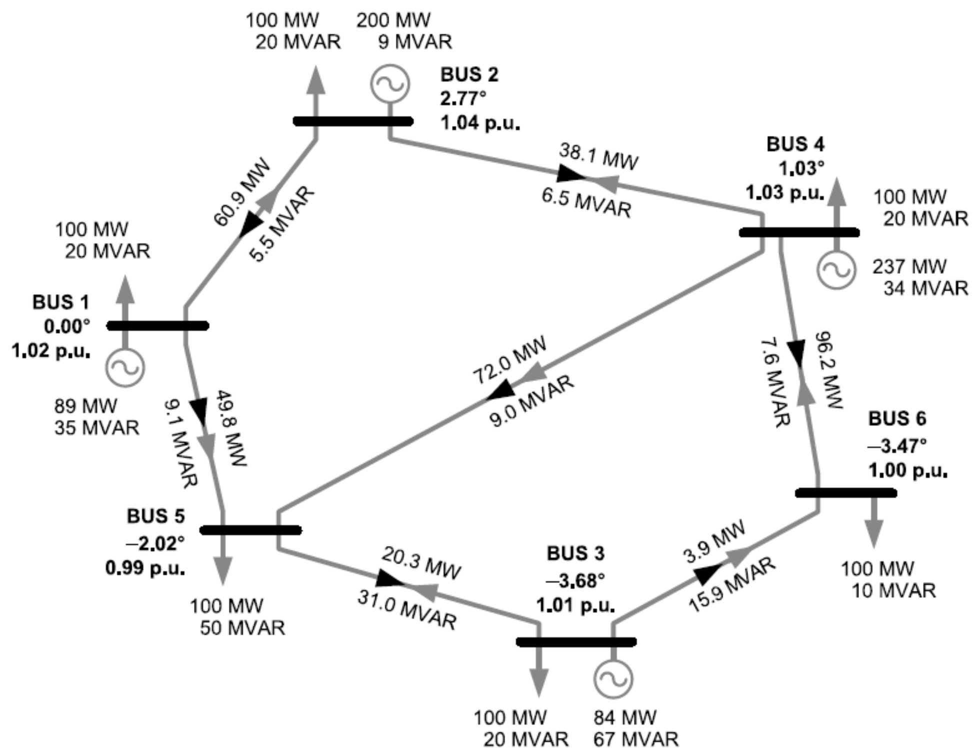
Remember that the power flow in an electrical transmission network is governed by the laws of physics, such as Kirchhoff's and Ohm's laws.

In fact, AC power flow equations are used to calculate the amount of power to be transferred across each transmission line.



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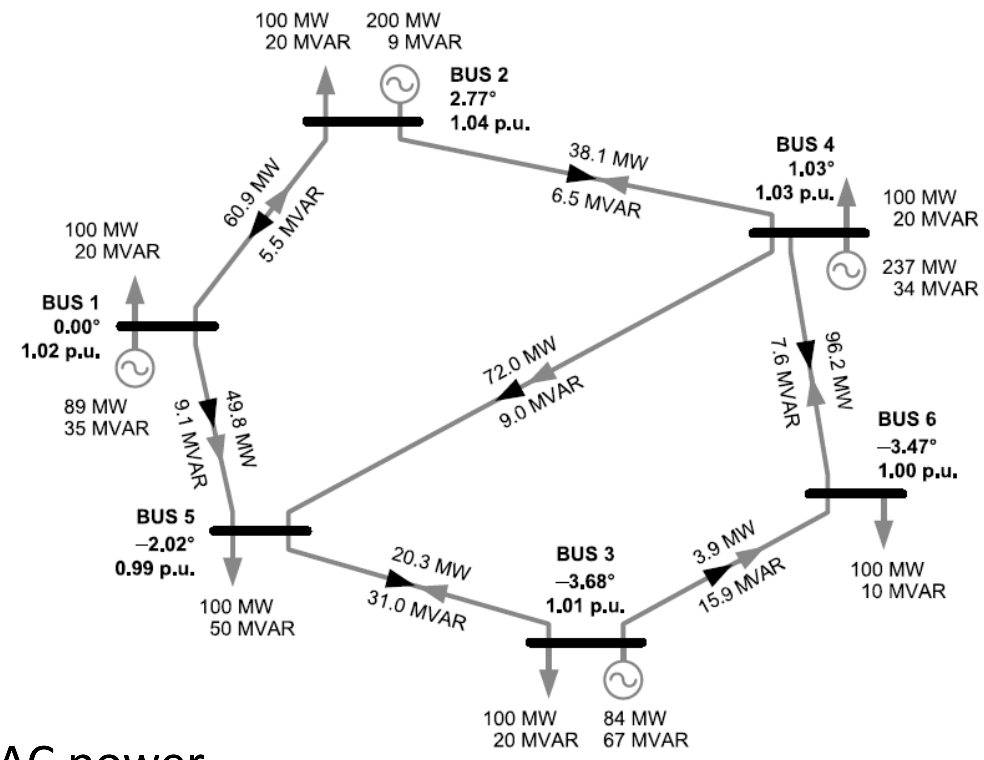

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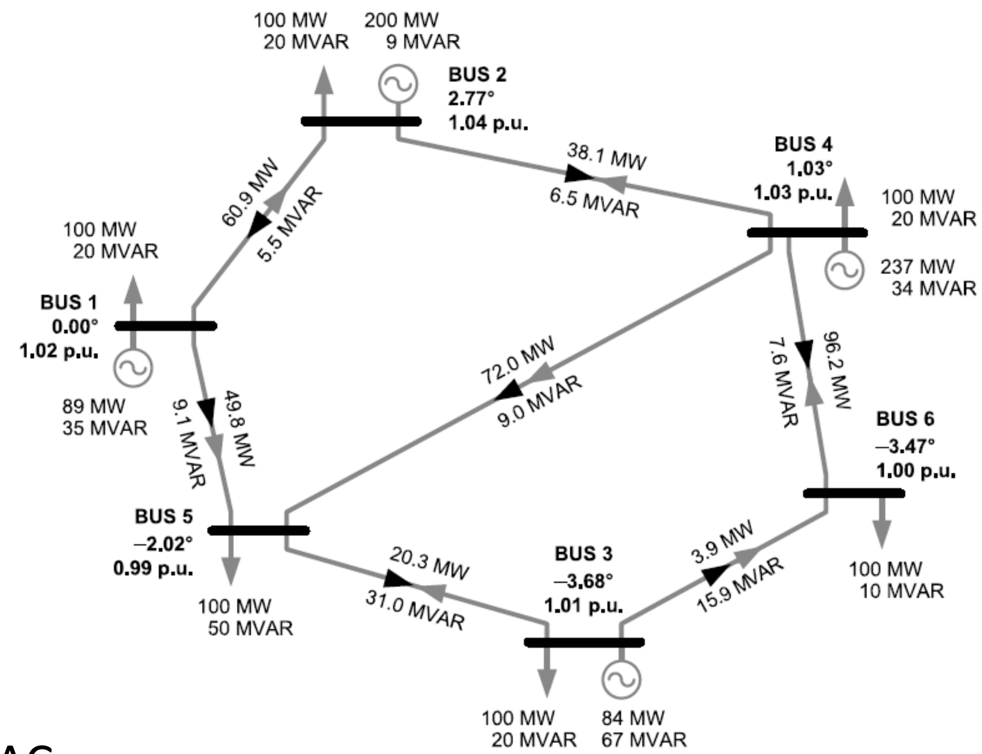
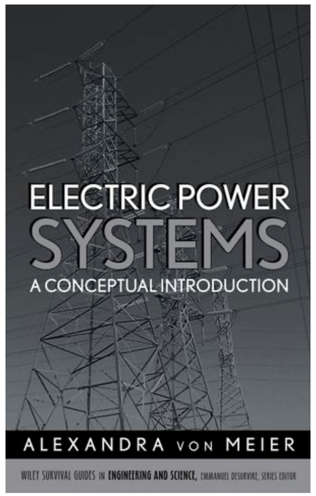
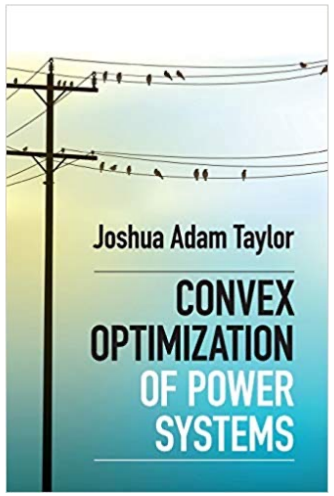


- Are you familiar with or do you recall the AC power flow equations?

- Among many other relevant books, you can learn about AC power flow equations from these two books:





AC power flow equations: Quick review

For every bus $i = \{1, 2, \ldots, n\}$, the following two equalities hold:

$$P_i = \sum_{k=1}^{n} |V_i||V_k|[g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|[g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k)]$$

Definition of symbols:

- Generated/consumed **active power** at each bus i, denoted by $\boldsymbol{P_i}$,
- Generated/consumed **reactive power** at each bus i, denoted by $\boldsymbol{Q_i}$,
- **Voltage magnitude** at each bus i, denoted by $\boldsymbol{V_i}$,
- **Voltage angle** at each bus i, denoted by $\boldsymbol{\theta_i}$,
- **Conductance** and **susceptance** of the line connecting buses i and k (input network data), denoted by $\boldsymbol{g_{i,k}}$ and $\boldsymbol{b_{i,k}}$.

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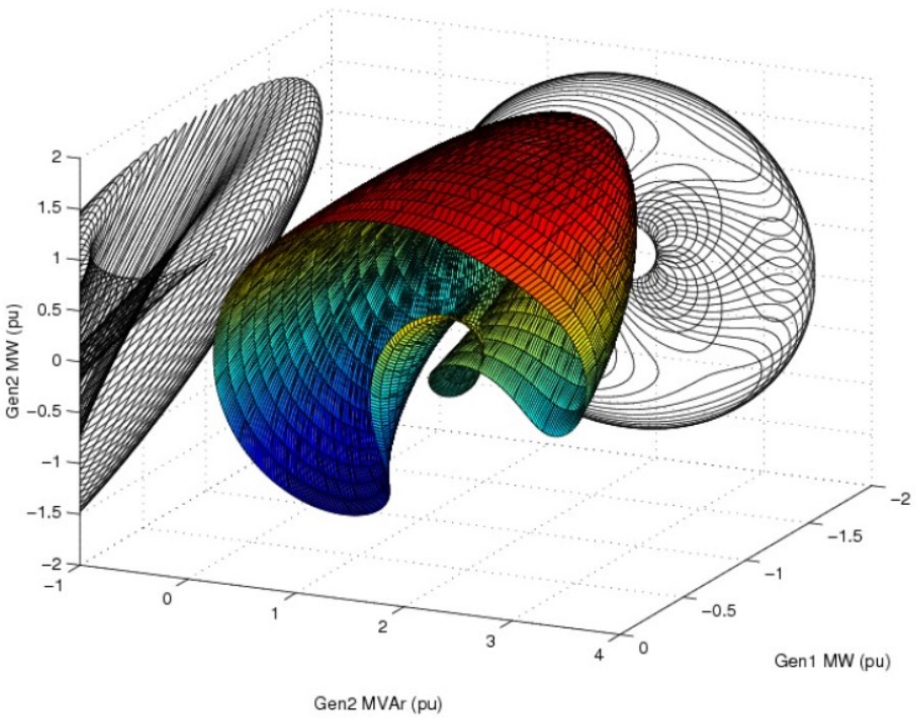
- Depending on the type of each bus i, two out of the four variables, $\mathbf{P_i}$, $\mathbf{Q_i}$, $\mathbf{V_i}$ and $\boldsymbol{\theta}_\mathbf{i}$, are known, while the other two are unknown.
- This ensures that the number of unknown variables and equations is balanced.

AC power flow equations: Quick review

However, the AC power flow equations are **non-linear** and **non-convex**, which adds complexity to solving them (NP-hard problem).

When incorporating these equations into an optimization problem, there is **no** guarantee of obtaining the **global** optimal solution.



An example of the feasible space in an AC optimal power flow problem (the feasible area for sample 3 variables limited by power flow constraints).

Credit: Ian Hiskens et al.

Power flow equations in the market-clearing problem

Our recent example of the market-clearing optimization problem with two buses and one transmission line

$$\underset{p^{\mathrm{D1}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}},p^{1\to2}}{\text{Maximize}} \quad 40\ p^{\mathrm{D1}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}$$

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Power flow constraints

$$p^{1\to2} - p^{\mathrm{W1}} - p^{\mathrm{G1}} = 0 \quad : \quad \lambda^{\mathrm{Bus1}}$$

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Shall we incorporate the AC power flow equations into the market-clearing optimization problem as power flow constraints? What challenges would arise?

Power flow equations in the market-clearing problem

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Shall we incorporate the AC power flow equations into the market-clearing optimization problem as power flow constraints? What challenges would arise?

One can see the day-ahead market as a financial market (in the day-ahead stage, though), providing some “contracts” for buying and selling energy. The actual power flow occurs during real-time operation.

Linearized AC power flow approximation

Also, referred to as "DC power flow approximation":

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Also, referred to as “DC power flow approximation”:

$$P_i = \sum_{k=1}^{n} |V_i||V_k|[g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|[g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k)]$$

Linearized AC power flow approximation

Also, referred to as “DC power flow approximation”:

1 per-unit

1 per-unit

$$P_i = \sum_{k=1}^{n} |\cancel{V_i}||\cancel{V_k}|[g_{ik}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|[g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k)]$$

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Also, referred to as "DC power flow approximation":

$$P_i = \sum_{k=1}^{n} \overset{\text{1 per-unit}}{|V_i|}\,\overset{\text{1 per-unit}}{|V_k|}[\overset{0}{g_{ik}}\cos(\theta_i - \theta_k) + b_{ik}\sin(\theta_i - \theta_k)]$$

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$$\cancelto{0}{Q_i} = \sum_{k=1}^{n} |V_i||V_k|[g_{ik}\sin(\theta_i - \theta_k) - b_{ik}\cos(\theta_i - \theta_k)]$$

$$P_i = \sum_{k=1}^{n} b_{ik}\,(\theta_i - \theta_k)$$

Recall symbols:

- Generated/consumed **active power** at each bus i, denoted by $\boldsymbol{P_i}$,
- **Voltage angle** at each bus i, denoted by $\boldsymbol{\theta_i}$,
- **susceptance** of the line connecting buses i and k (input network data), denoted by $\boldsymbol{b_{i,k}}$.

Linearized AC power flow approximation

Also, referred to as “DC power flow approximation”:

SHORT-PAPER | OPEN ACCESS

Solutions of DC OPF are Never AC Feasible

Author: Kyri Baker | Authors Info & Claims

e-Energy '21: Proceedings of the Twelfth ACM International Conference on Future Energy Systems • Pages 264 - 268
https://doi.org/10.1145/3447555.3464875

Published: 22 June 2021 Publication History

Check for updates

https://dl.acm.org/doi/abs/10.1145/3447555.3464875

Power flow equations in the market-clearing problem

Our recent example of the market-clearing optimization problem with two buses and one transmission line

$$\underset{p^{\mathrm{D1}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}},p^{1\to2}}{\text{Maximize}} \quad 40\ p^{\mathrm{D1}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$0 \leq p^{\mathrm{D1}} \leq 90 \quad : \quad \underline{\mu}^{\mathrm{D1}}, \overline{\mu}^{\mathrm{D1}}$$

$$\boxed{-60 \leq p^{1\to2} \leq 60 \quad : \quad \underline{\mu}^{1\to2}, \overline{\mu}^{1\to2}}$$

Power flow constraints

$$p^{1\to2} - p^{\mathrm{W1}} - p^{\mathrm{G1}} = 0 \quad : \quad \lambda^{\mathrm{Bus1}}$$

$$p^{\mathrm{D1}} - p^{1\to2} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda^{\mathrm{Bus2}}$$

Linearized power flow equations

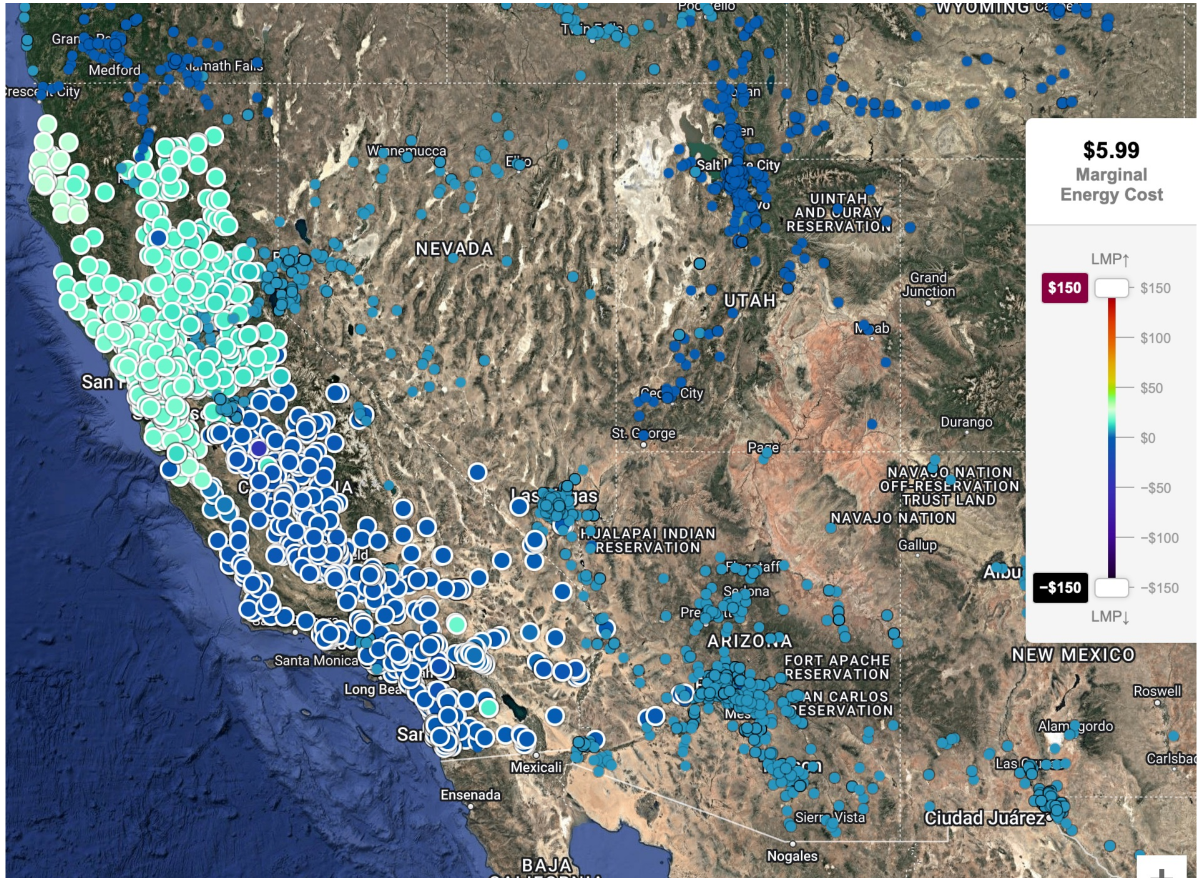
$$P_i = \sum_{k=1}^{n} b_{ik}\left(\theta_i - \theta_k\right)$$

Outline

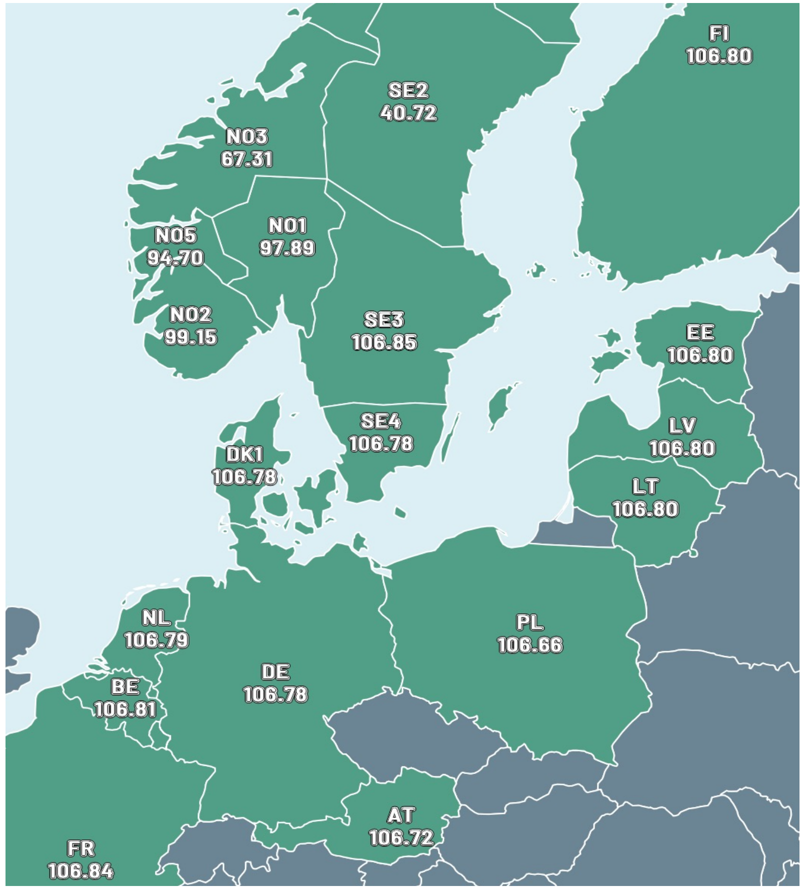
- Fundamentals of Day-Ahead Market Clearing
- Single Day-Ahead Coupling (SDAC) in Europe
- Network Effects on Market-Clearing Outcomes
- Power Flow Equations
- Market-Clearing Optimization Problem: Nodal vs. Zonal
- Flow-Based Market Coupling in Europe

Nodal vs. zonal electricity markets

Nodal prices (in $/MWh) in **CAISO** (California, U.S.)
February 16, 2025, 10-11pm ([link]())



Zonal prices (in €/MWh) in **Europe**
February 17, 2025, 1-2pm ([link]())



Nodal vs. zonal electricity markets

- Each bidding zone in Europe has a single price (e.g., one price for the entire Germany).
- Transmission line capacity limits within bidding zones are **not** enforced in the market-clearing optimization problem.
- Price differences between two zones occur only when power trade between them reaches the net transfer capacity (NTC) set by the TSOs.

Zonal prices (in €/MWh) in **Europe**
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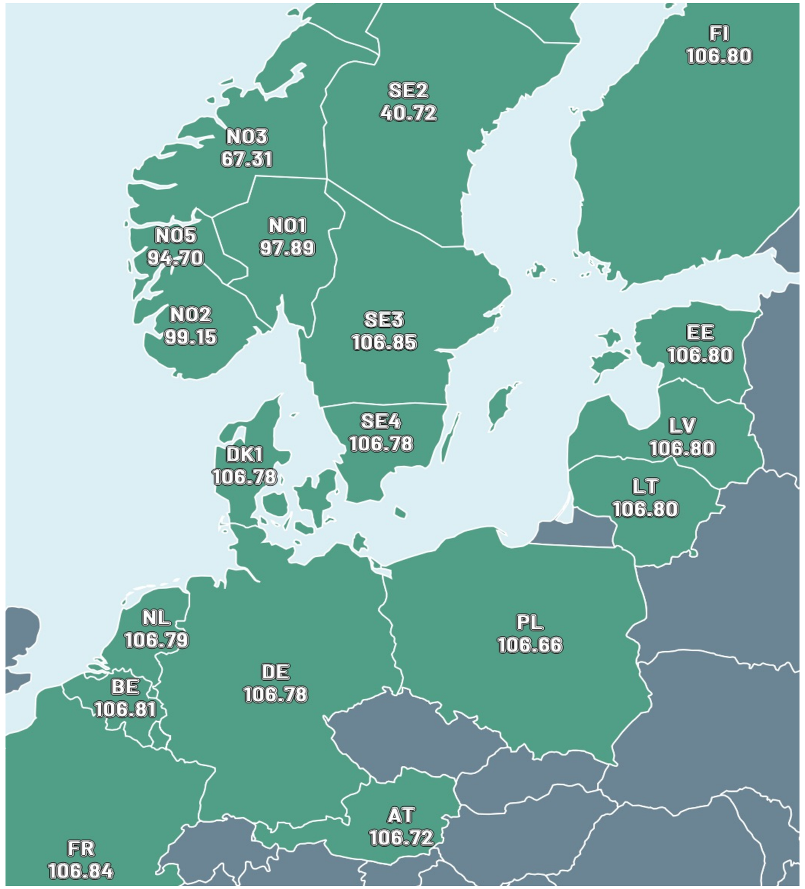

Nodal vs. zonal electricity markets

The limits of critical transmission lines are enforced to some extent in the "*flow-based coupling*" approach. This will be discussed in the next slides.

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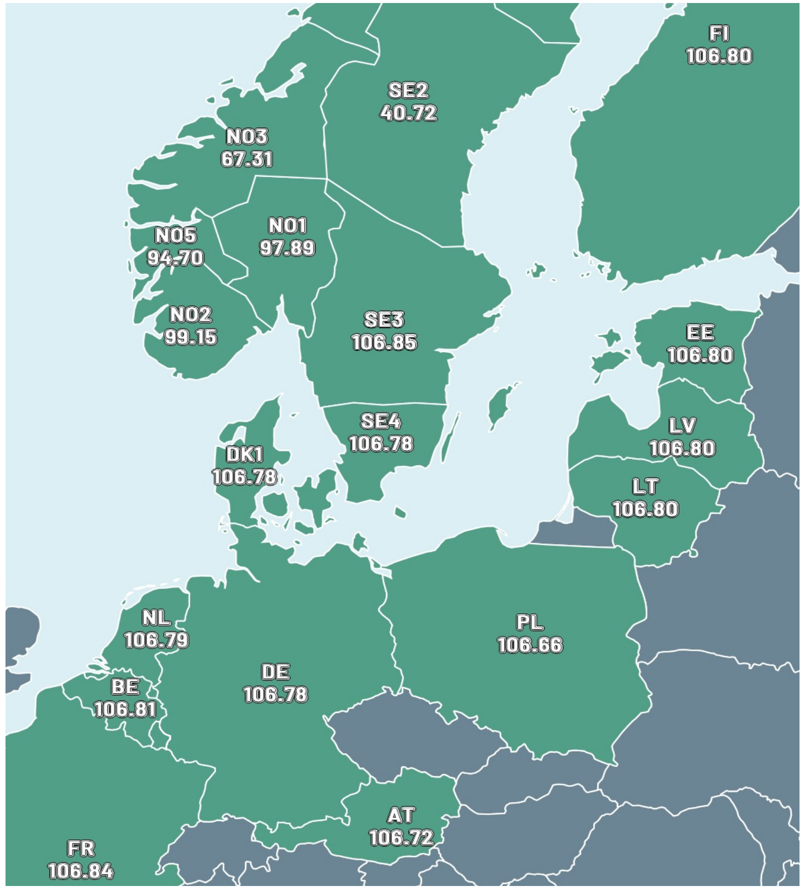

Nodal vs. zonal electricity markets

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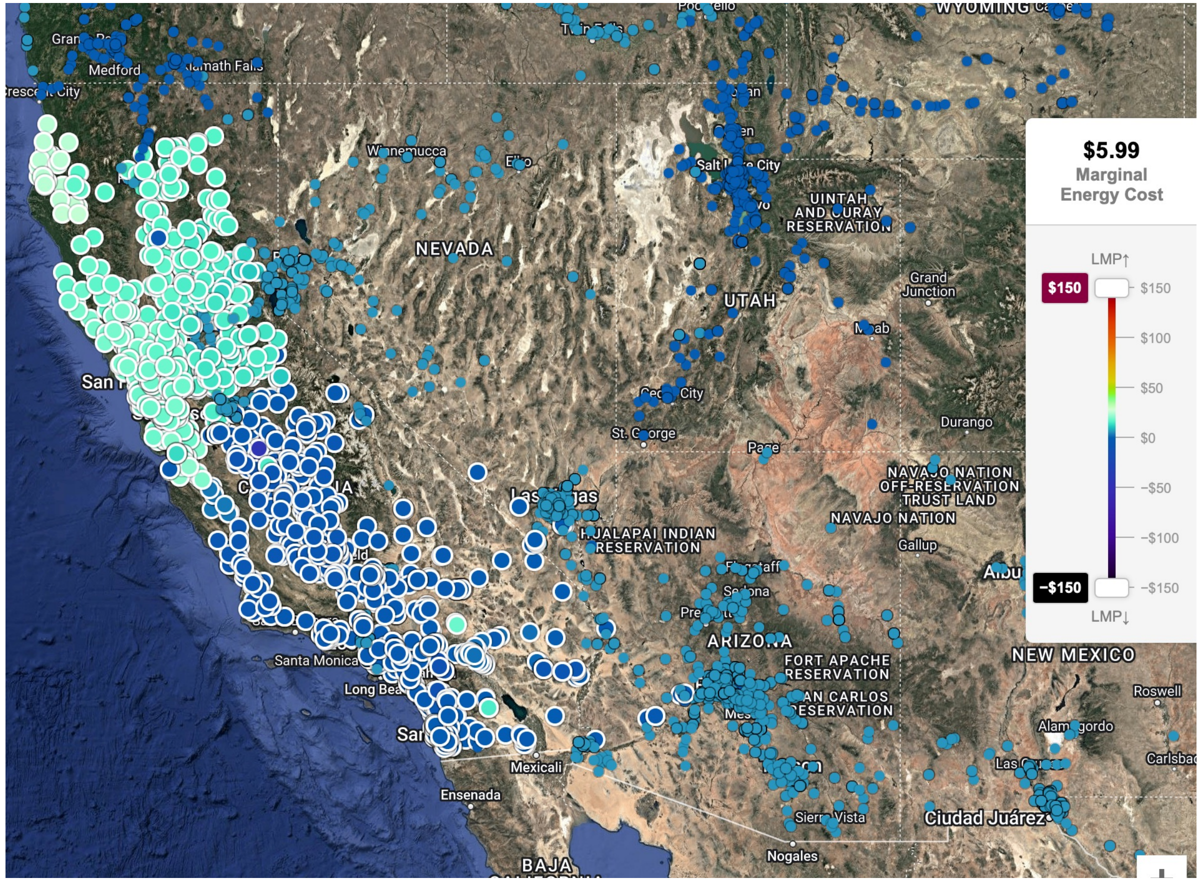


- ISOs in the U.S. use linearized power flow equations to approximate power flow and enforce transmission line capacity limits in the market-clearing optimization problem.
- As a result, nodal prices—also referred to as **locational marginal prices (LMPs)**—are determined, meaning there is one price per bus (node).

Nodal vs. zonal electricity markets

Redispatch Cost:

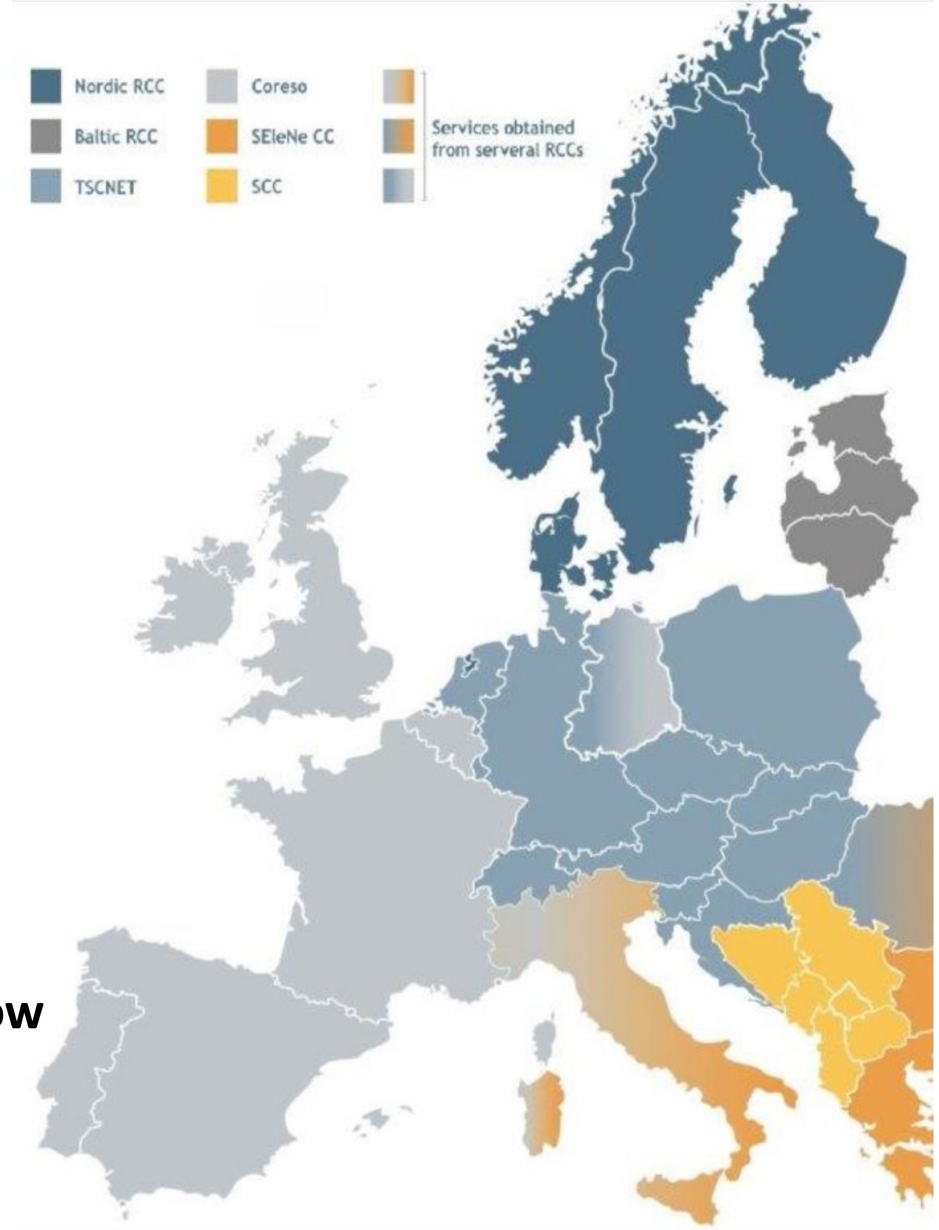
- After market clearing, TSOs request every market participant with an accepted bid/offer to provide a detailed production/consumption plan within zones.
- TSOs then simulate whether market outcomes are **feasible** concerning transmission network limits.
- If infeasibilities arise, **redispatch** is required!
- German TSOs, in particular, incur **significant annual redispatch costs**—amounting to multiple billions of euros per year—due to expensive redispatch actions.

Source: Clean Energy Wire

Cross-border coordination in Europe

There are six Regional Coordination Centres (RCCs) across Europe:

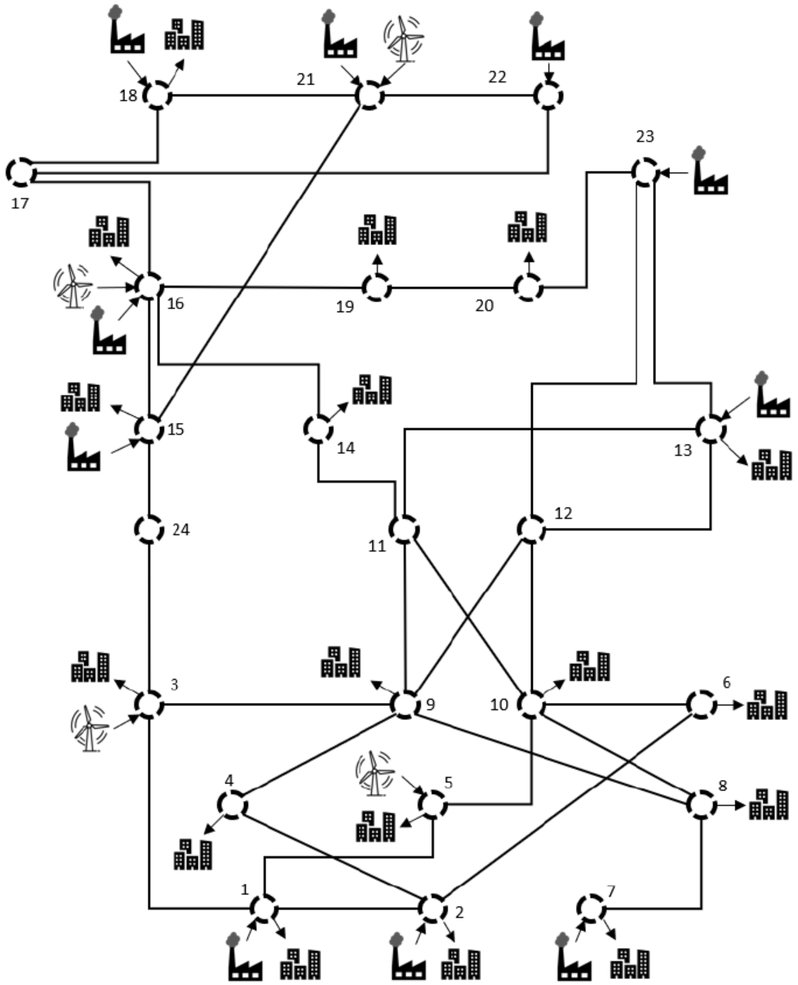
- **Nordic RCC** – based in Copenhagen
- **TSCNET Services** – based in Munich
- **Coreso** – based in Brussels
- **SCC** – based in Serbia
- **Baltic RSC** – served from the three national control centres
- **Selene CC** – based in Greece



Among other tasks, RCCs coordinate with TSOs on **NTCs (Net Transfer Capacities)** or **power flow coupling parameters** to enhance cross-border electricity exchanges and grid stability.

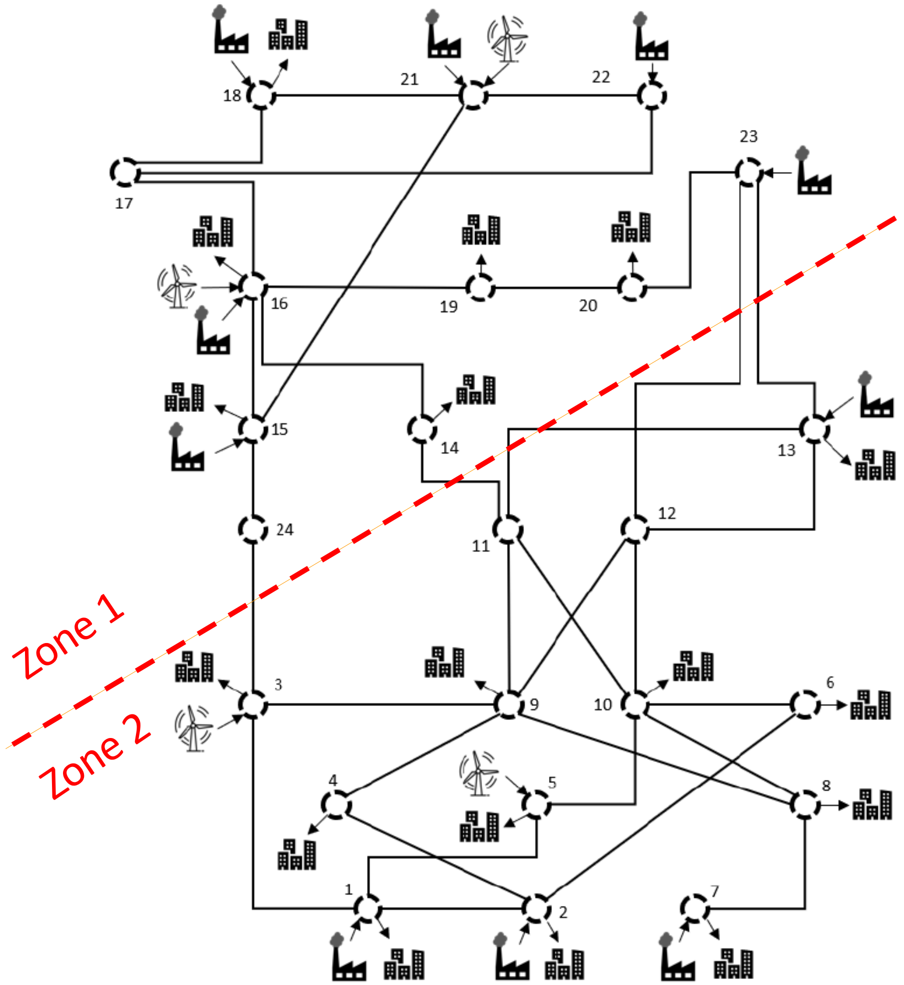
Market-clearing optimization: Nodal vs. zonal

A **nodal** system with 24 buses (nodes)

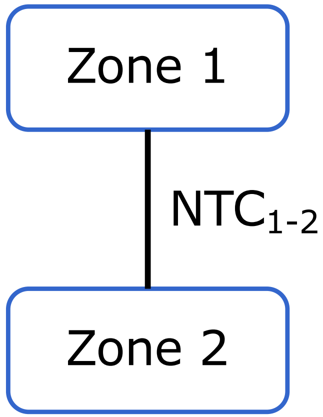


Market-clearing optimization: Nodal vs. zonal

Example: A **zonal** system with two zones

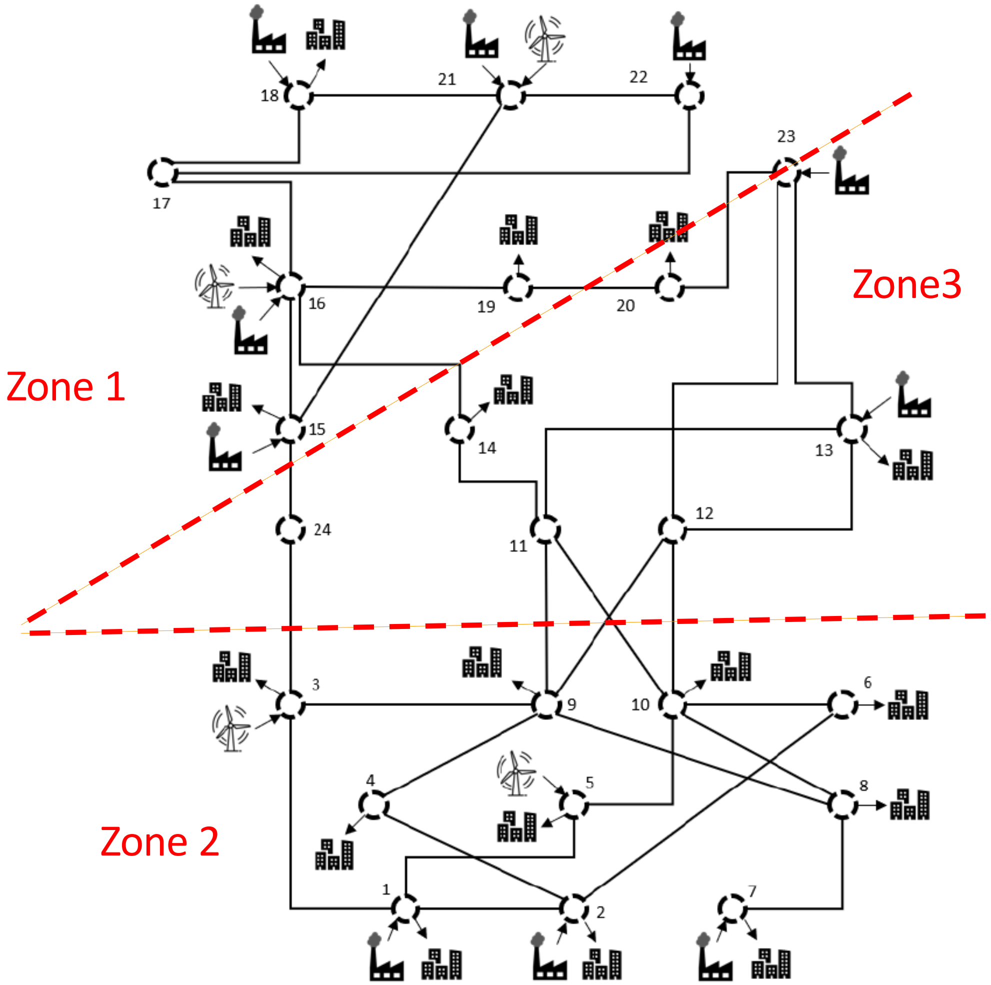


- Zone 1 includes nodes 1 to 13, while Zone 2 encompasses the remaining nodes.
- There is one NTC value between the two connecting zones.
- Transmission line limits within the zones will be disregarded.

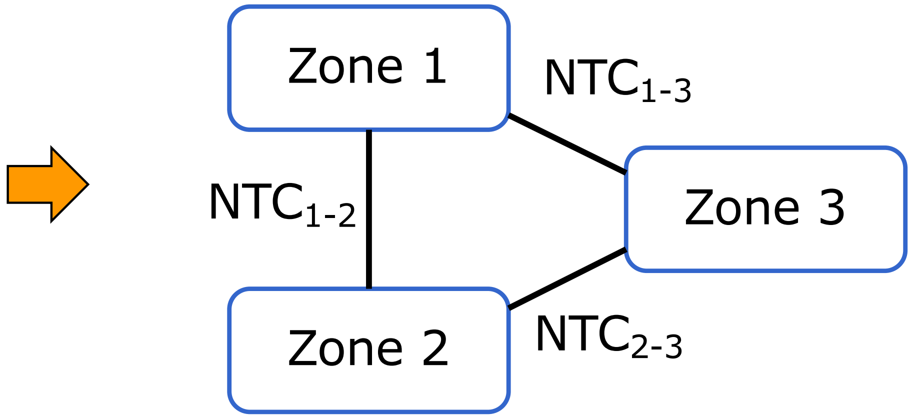


Market-clearing optimization: Nodal vs. zonal

Example: A **zonal** system with **three** zones



There are three NTC values, one for each pair of connecting zones.



Power flow in nodal systems

- Linearized power flow equations

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- While representing the power flow and transmission capacity limits, the resulting market-clearing optimization problem is linear and convex. Therefore, the solution obtained is proven to be globally optimal.

Power flow in nodal systems

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- While representing the power flow and transmission capacity limits, the resulting market-clearing optimization problem is linear and convex. Therefore, the solution obtained is proven to be globally optimal.
- Accordingly, the following equation gives the power flow across the transmission line connecting bus *n* to bus *m*:

$$f_{n,m} = B_{n,m}\underbrace{(\theta_n - \theta_m)}_{\text{Difference of voltage angles of buses } n \text{ and } m \text{ (variable)}}$$

Power flow from bus *n* to bus *m* (variable)

Susceptance of the line connecting buses *n* to *m* (given data)

Power flow in nodal systems

- Linearized power flow equations
- While representing the power flow and transmission capacity limits, the resulting market-clearing optimization problem is linear and convex. Therefore, the solution obtained is proven to be globally optimal.
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Power flow from bus *n* to bus *m* (variable)

Susceptance of the line connecting buses *n* to *m* (given data)

- Additionally, the following inequalities impose limits on the power flow across the transmission line connecting bus *n* to bus *m*:

$$-F_{n,m} \leq f_{n,m} \leq F_{n,m}$$

Capacity of the line connecting buses *n* to *m* (given data)

Market-clearing optimization: Nodal

Compact form:

Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}},p_d^{\mathrm{D}},\theta_n}{\text{Maximize}} \quad SW = \sum_d U_d\, p_d^{\mathrm{D}} - \sum_g C_g\, p_g^{\mathrm{G}}$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g$$

$$\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n$$

$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

Market-clearing optimization: Nodal

Compact form:

$$\underset{\boxed{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

Set of primal variables

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g$$

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$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

Market-clearing optimization: Nodal

Compact form:

Bid price of demand d

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \; SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

Social welfare

Offer price of producer g

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g$$

$$\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n$$

$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

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Market-clearing optimization: Nodal

Compact form:

Consumption of demand *d*

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

Production of producer *g*

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g$$

$$\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n$$

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Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$ → **Consumption limits of demand *d***

$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g$$ → **Production limits of producer *g***

$$\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n$$

$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

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Market-clearing optimization: Nodal

Compact form:

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Power balance at bus *n*

$$\boxed{\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n}$$

$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

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Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

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$$0 \le p_g^{\mathrm{G}} \le \overline{P}_g^{\mathrm{G}} \quad \forall g$$

This term represents the total power ***leaving*** bus *n*, so it can be interpreted as demand, having the same sign as demand (first term).

$$\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \boxed{\sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m)} - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n$$

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$$\theta_{ref} = 0$$

Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

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$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

All demands located at bus *n*

Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

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$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

All buses *m* connected to bus *n* through transmission lines

Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

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$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

All producers located at bus *n*

Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}},p_d^{\mathrm{D}},\theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

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$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

Nodal price at bus *n*
(locational marginal price, LMP)

Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

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$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g$$

$$\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n$$

$$\boxed{-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m}$$

$$\theta_{ref} = 0$$

Transmission limits of the line connecting bus *n* to bus *m*

Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}},p_d^{\mathrm{D}},\theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g$$

$$\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n$$

$$-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

Capacity of the line connecting bus *n* to bus *m*

Market-clearing optimization: Nodal

Compact form:

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, \theta_n}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

subject to:

$$0 \le p_d^{\mathrm{D}} \le \overline{P}_d^{\mathrm{D}} \quad \forall d$$

$$0 \le p_g^{\mathrm{G}} \le \overline{P}_g^{\mathrm{G}} \quad \forall g$$

$$\sum_{d \in \Psi_n} p_d^{\mathrm{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\mathrm{G}} = 0 \qquad : \lambda_n \quad \forall n$$

$$-F_{n,m} \le B_{n,m}(\theta_n - \theta_m) \le F_{n,m} \quad \forall n, \forall m$$

$$\theta_{ref} = 0$$

Voltage angle at the reference bus (e.g., bus 1)

Without this constraint, the number of variables exceeds the number of equations, potentially leading to multiple solutions for the voltage angles!

Power flow in zonal systems

- Neither AC nor linearized power flow equations are included.

Power flow in zonal systems

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- Power trade between each pair of connecting zones is restricted to NTCs (Net Transfer Capacities), as determined by TSOs (or RCCs).

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- The day-ahead power trade between zones aa and bb is limited by:

$$-ATC_{a,b} \leq f_{a,b} \leq ATC_{a,b}$$

Power transfer from zone *a* to zone *b* (variable)

NTC between zones *a* and *b* (given parameter)

Disclaimer: I use the symbol ATC (**Available** Transfer Capacity) to represent NTC (**Net** Transfer Capacity), although they might differ slightly in practice. In this course, we consider both terms (NTC and ATC) identical.

Power flow in <u>zonal</u> systems

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Power transfer from zone *a* to zone *b* (variable)

NTC between zones *a* and *b* (given parameter)

Examples of NTCs set by TSOs and the corresponding RCC:

Symmetric NTCs: $-2000 \leq f_{a,b} \leq 2000$ ⇨ Trade between zone *a* and zone *b* (in either direction) is limited to a maximum of 2000 MW.

Power flow in zonal systems

- Neither AC nor linearized power flow equations are included.
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$$-ATC_{a,b} \leq f_{a,b} \leq ATC_{a,b}$$

Power transfer from zone *a* to zone *b* (variable)

NTC between zones *a* and *b* (given parameter)

Examples of NTCs set by TSOs and the corresponding RCC:

Asymmetric NTCs: $-1200 \leq f_{a,b} \leq 2000$ ⇒ Trade from zone *a* to zone *b* is limited to a maximum of 2000 MW, while trade in the opposite direction is limited to 1200 MW.

Market-clearing optimization: Zonal

How to transform the nodal problem into a zonal problem?

Market-clearing optimization: Zonal

Nodal → Zonal

$$\underset{p_g^{\text{G}}, p_d^{\text{D}}, \cancel{\theta_n}}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\text{D}} - \sum_g C_g \, p_g^{\text{G}}$$

subject to:

$$0 \leq p_d^{\text{D}} \leq \overline{P}_d^{\text{D}} \quad \forall d$$

$$0 \leq p_g^{\text{G}} \leq \overline{P}_g^{\text{G}} \quad \forall g$$

$$\cancel{\sum_{d \in \Psi_n} p_d^{\text{D}} + \sum_{m \in \Omega_n} B_{n,m}(\theta_n - \theta_m) - \sum_{g \in \Psi_n} p_g^{\text{G}} = 0 \quad : \lambda_n \quad \forall n}$$

$$\cancel{-F_{n,m} \leq B_{n,m}(\theta_n - \theta_m) \leq F_{n,m} \quad \forall n, \forall m}$$

$$\cancel{\theta_{ref} = 0}$$

We remove this set of constraints and instead incorporate the NTC-based constraints (next slide).

Market-clearing optimization: Zonal

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, f_{a,b}}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

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$$\boxed{\begin{aligned} &\sum_{d \in \Psi_a} p_d^{\mathrm{D}} + \sum_{b \in \Omega_a} f_{a,b} - \sum_{g \in \Psi_a} p_g^{\mathrm{G}} = 0 \qquad : \lambda_a^{\mathrm{Zonal}} \quad \forall a \\ &-ATC_{a,b} \leq f_{a,b} \leq ATC_{a,b} \qquad \forall a, \forall b \in \Omega_a \end{aligned}}$$

Market-clearing optimization: Zonal

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, f_{a,b}}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

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$$-ATC_{a,b} \leq f_{a,b} \leq ATC_{a,b} \qquad \forall a, \forall b \in \Omega_a$$

All demands
located at zone *a*

Market-clearing optimization: Zonal

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, f_{a,b}}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

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$$- ATC_{a,b} \leq f_{a,b} \leq ATC_{a,b} \qquad \forall a, \forall b \in \Omega_a$$

All zones *b* connected to zone *a*

Market-clearing optimization: Zonal

$$\underset{p_g^{\mathrm{G}},p_d^{\mathrm{D}},f_{a,b}}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

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$$\sum_{d \in \Psi_a} p_d^{\mathrm{D}} + \sum_{b \in \Omega_a} f_{a,b} - \sum_{g \in \Psi_a} p_g^{\mathrm{G}} = 0 \qquad : \lambda_a^{\mathrm{Zonal}} \qquad \forall a$$

$$-ATC_{a,b} \le f_{a,b} \le ATC_{a,b} \qquad \forall a, \forall b \in \Omega_a$$

All producers located at zone *a*

Market-clearing optimization: Zonal

$$\underset{p_g^{\mathrm{G}}, p_d^{\mathrm{D}}, f_{a,b}}{\text{Maximize}} \quad SW = \sum_d U_d \, p_d^{\mathrm{D}} - \sum_g C_g \, p_g^{\mathrm{G}}$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d$$

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$$\sum_{d \in \Psi_a} p_d^{\mathrm{D}} + \sum_{b \in \Omega_a} f_{a,b} - \sum_{g \in \Psi_a} p_g^{\mathrm{G}} = 0 \qquad : \lambda_a^{\mathrm{Zonal}} \quad \forall a$$

$$-ATC_{a,b} \leq f_{a,b} \leq ATC_{a,b} \qquad \forall a, \forall b \in \Omega_a$$

Zonal price at zone *a*

Outline

- Fundamentals of Day-Ahead Market Clearing
- Single Day-Ahead Coupling (SDAC) in Europe
- Network Effects on Market-Clearing Outcomes
- Power Flow Equations
- Market-Clearing Optimization Problem: Nodal vs. Zonal
- Flow-Based Market Coupling in Europe

Flow-based market coupling in Europe

- It is an alternative to the NTC-based zonal model.

- The Central Western European (**CWE**) TSOs successfully implemented flow-based capacity calculation and allocation, with the go-live date on **May 21st, 2015**.

- Flow-based market coupling in the **Nordics** went live on **October 29th, 2024** (energy delivery day: October 30th).

Flow-based market coupling in Europe

Objective

- Improve the efficiency of NTC-based zonal market clearing by incorporating network constraints of critical domestic (intra-zonal) lines.

Flow-based market coupling in Europe

Objective

- Improve the efficiency of NTC-based zonal market clearing by incorporating network constraints of critical domestic (intra-zonal) lines.
- Retain the zonal structure, ensuring a single price for each bidding zone.

Flow-based market coupling in Europe

Objective

- Improve the efficiency of NTC-based zonal market clearing by incorporating network constraints of critical domestic (intra-zonal) lines.
- Retain the zonal structure, ensuring a single price for each bidding zone.
- Maintain the market-clearing optimization problem as linear and convex.

Flow-based market coupling in Europe

Objective

- Improve the efficiency of NTC-based zonal market clearing by incorporating network constraints of critical domestic (intra-zonal) lines.
- Retain the zonal structure, ensuring a single price for each bidding zone.
- Maintain the market-clearing optimization problem as linear and convex.

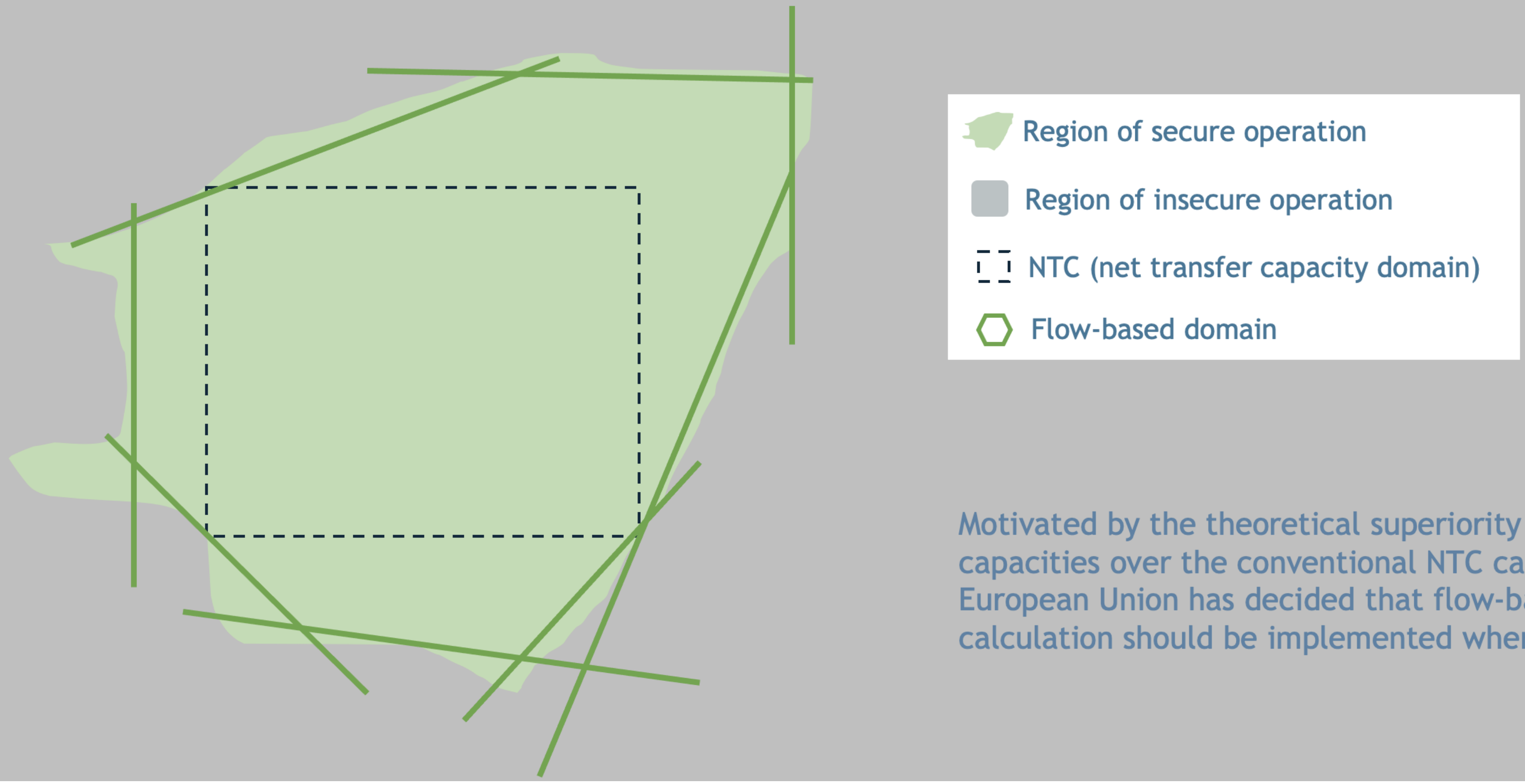
This is achieved by incorporating polyhedral constraints on the zonal net injections within the market-clearing optimization model.

However, it remains far from the complexity of the nodal model.

Flow-based market coupling in Europe

VALUE OF FLOW-BASED MARKET COUPLING

Theory: Flow-based domain can be a better approximation of the region of secure operation and give the market more freedom in optimizing socio-economic welfare.



Motivated by the theoretical superiority of flow-based capacities over the conventional NTC capacities European Union has decided that flow-based capacity calculation should be implemented where applicable.

Credit: Jakob Glarbo Møller (Nordic RCC, Copenhagen)

Flow-based market coupling in Europe

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Flow-based constraints

What are flow-based constraints?

Flow-based capacity constraints represent limits on the critical network elements in the power grid.

Each constraint is represented as a row in the matrix, and it contains the following values:

- **RAM (Remaining Available Margin)** - the available margin for the critical network element, which serves as an upper bound on the total consumption of this element by different bidding areas.
- A list of **PTDFs (Power Transfer Distribution Factors)** for each Bidding Zone in the corresponding Flow-Based region. A PTDF indicates how much a bidding zone's Net-Position contributes to the load of the critical network element (limited by RAM).

To see if a constraint is binding (or how close it is to being reached), multiply the **Net-Position (NP)** for each Bidding Zone by its respective PTDF. Then, sum the results of each multiplication. This must result in a value that is lower or equal to the RAM.
When the sum equals the RAM, the **corresponding constraint (CNEC)** is binding. This can be exemplified by the following formula:
$PTDF1 * NP1 + PTDF2 * NP2 + \ldots + PTDFn * NPn \leq RAM$

Euphemia ensures that these constraints will always be respected. Flow-based capacity constraints are used in Flow-Based regions.

The **Net-Position** of a Bidding Zone is defined as:
Total Sell Volume - Total Buy Volume
or
Total Export - Total Import for the Bidding Zone.

The total sell/buy volume in a Bidding Zone is the sum of the traded volumes from every **NEMO (Nominated Electricity Market Operator)** operating in that zone. Therefore, the data from our volume views only account for our members and cannot be considered as total market volumes in Bidding Zones where Nord Pool is not the sole NEMO.

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Flow-based capacity constraints represent limits on the critical network elements in the power grid.

This could be a domestic transmission line in a certain zone, e.g., DK2.

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Flow-based constraints

What are flow-based constraints?

For example, if the RAM is 100 MW, it is provided as input data to the market-clearing optimization problem.

Flow-based capacity constraints represent limits on the critical network elements in the power grid.

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PTDFs are provided as input data, derived from linearized power flow equations.

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Euphemia ensures that these constraints will always be respected. Flow-based capacity constraints are used in Flow-Based regions.

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When the sum equals the RAM, the **corresponding constraint (CNEC)** is binding. This can be exemplified by the following formula:

$$\boxed{PTDF1 * NP1 + PTDF2 * NP2 + \ldots + PTDFn * NPn \leq RAM}$$

One linear constraint is applied for each critical line.

Euphemia ensures that these constraints will always be respected. Flow-based capacity constraints are used in Flow-Based regions.

The **Net-Position** of a Bidding Zone is defined as:
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The **Net-Position** of a Bidding Zone is defined as:
Total Sell Volume - Total Buy Volume
or
Total Export - Total Import for the Bidding Zone.

NP for each bidding zone is now treated as a variable in the market-clearing optimization problem.

The total sell/buy volume in a Bidding Zone is the sum of the traded volumes from every **NEMO (Nominated Electricity Market Operator)** operating in that zone. Therefore, the data from our volume views only account for our members and cannot be considered as total market volumes in Bidding Zones where Nord Pool is not the sole NEMO.

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Flow-based market coupling in Europe

Input data for the market-clearing optimization problem with flow-based market coupling:

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Domains: CORE | NORDIC | DK1

Each row corresponds to a certain critical line

Remaining Available Margin (MW) (RAM)	DK2 (PTDFs)	FI	NO1	NO2	SE1	NO4
1 000,0	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
489,0	-0,244820	-0,006770	-0,006690	-0,006690	-0,006770	-0,006750
253,0	-0,076570	-0,001460	-0,001450	-0,001450	-0,001460	-0,001460
236,0	0,115850	-0,000520	-0,000520	-0,000520	-0,000520	-0,000520
1 572,0	0,001330	0,410900	0,463270	0,464200	0,410970	0,420240
2 152,0	0,000340	0,283590	0,547870	0,552820	0,283970	0,326330
1 090,0	0,000160	0,147450	-0,078320	-0,084110	0,147330	0,128160
1 762,0	0,000130	0,267710	-0,090180	-0,089790	0,265800	0,139320
358,0	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
358,0	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
1 200,0	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
1 200,0	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
1 000,0	0,000000	1,000000	0,000000	0,000000	0,000000	0,000000
1 158,0	0,000000	-1,000000	0,000000	0,000000	0,000000	0,000000
1 200,0	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
7 483,0	-0,000050	0,924460	0,162590	0,145420	0,923620	0,833810

...

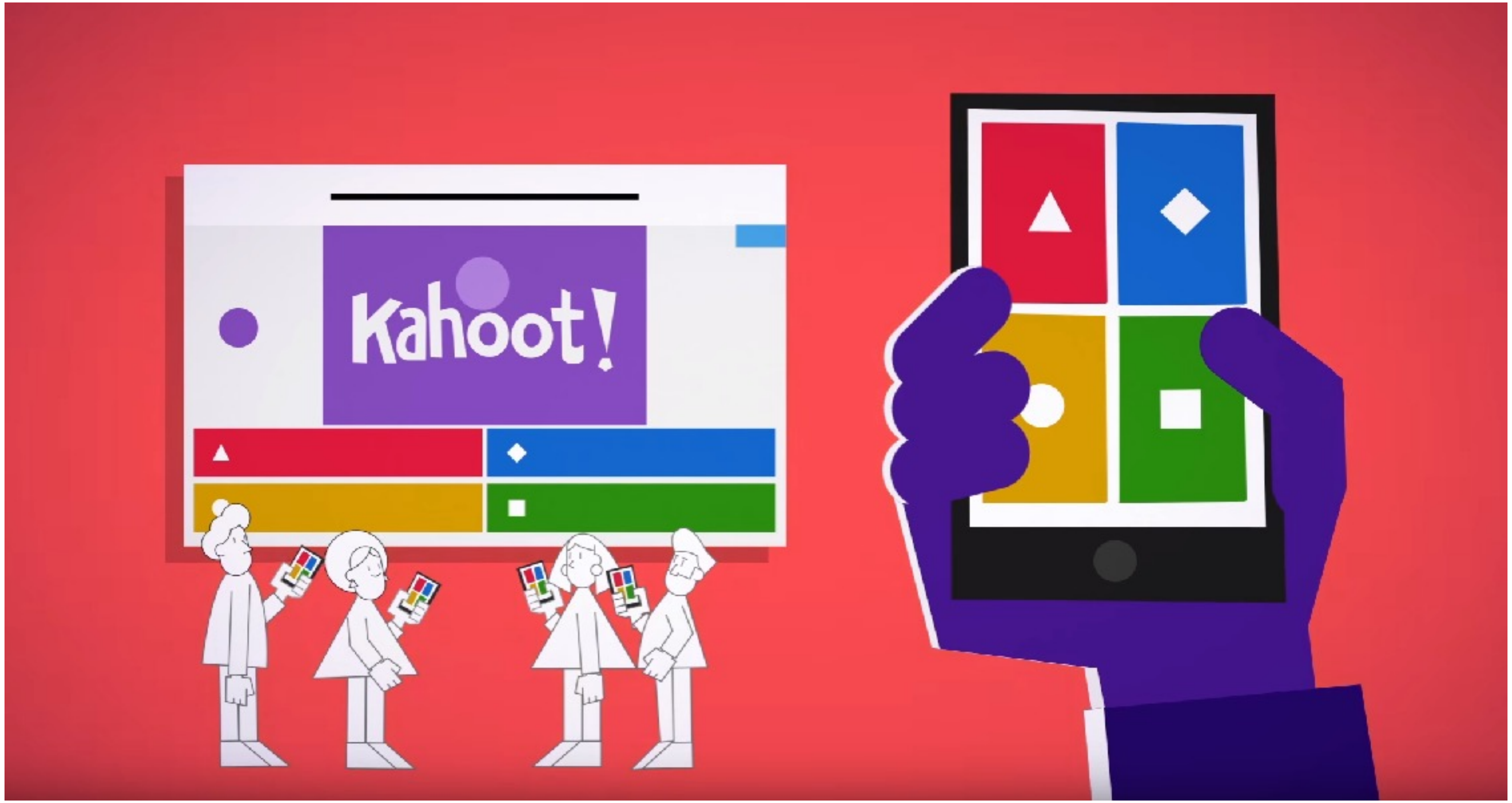
https://data.nordpoolgroup.com/auction/day-ahead/constraints?deliveryDate=2025-02-17&domain=NORDIC

Assignment 1

Full assignment is uploaded on DTU Learn and my webpage:
https://www.jalalkazempour.com/teaching

Your tasks:

- Select a case study and complete it by finding data from the data sources introduced.
- **Step 1: Copper-Plate, Single Hour** (you can work on it after Lecture 2)
- **Step 2: Copper-Plate, Multiple Hours** (you can work on it after Lecture 2)
- **Step 3: Network Constraints** (you can work on it after Lecture 3)
- **Step 4: Optimization vs. Equilibrium** (you can work on it after Lecture 4)
- **Step 5: Balancing Market** (you can work on it after Lecture 5)
- **Step 6: Reserve Market** (you can work on it after Lecture 6)


Kahoot!

Thanks for your attention!

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