Lecture 2 - Fundamentals of Electricity Market

Objectives:

- Identify the key factors in the electricity market.
- Describe the different types of markets in the power sector.
- Compare and contrast the current practices in European and U.S. electricity markets.
- Formulate the market-clearing procedure as a linear optimization problem, excluding the transmission network.

Outline

- Various market actors
- Various types of electricity market
- European vs U.S.electricity markets
- Market-clearing as a linear optimisation problem

1. Market actors

- Also referred to as “market players” or “market participants”
- ! Which market actors have we identified so far?

Power demands:

- Large consumers, e.g., industrial plants
- Retailers: Buy and resell electricity to smaller consumers.
  A retailer is an intermediate market actor (or trader) who purchases electricity in bulk from electricity markets and sells it to a large number of small-scale consumers, e.g., households.

Examples in Denmark:

Market operator

A non-profit entity that receives all offers from producers and bids from consumers - clears the market by maximizing social welfare, and ultimately distributes the market-clearing outcomes, including prices and quantities.

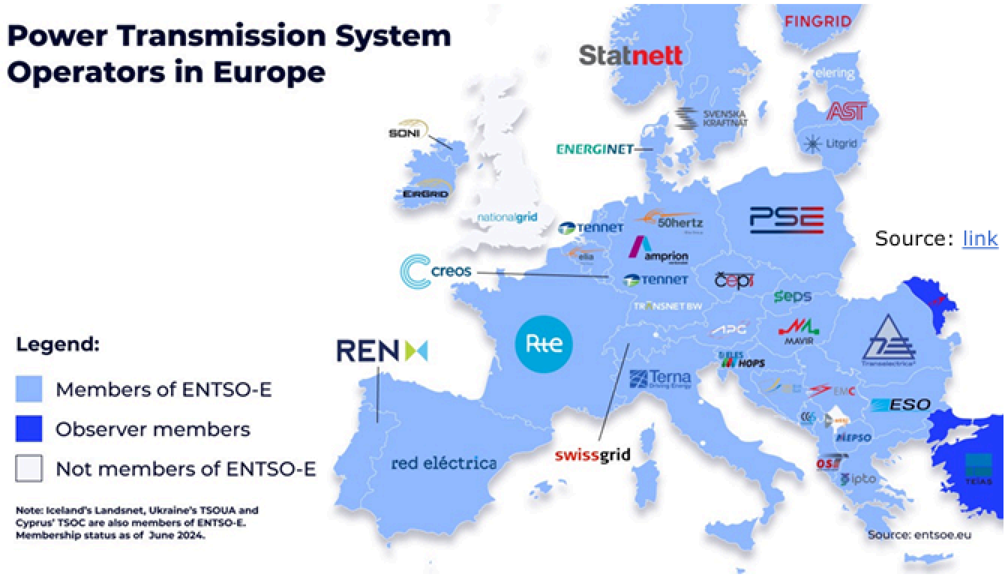
Examples in Europe:

System operator

- Transmission System Operator (TSO): Responsible for the safe operation of the underlying power grid at the high-voltage transmission level (typically a meshed grid), ensuring supply security, real-time power supply and demand balance, and system stability.
  - Example in Denmark:

    In Denmark, **Energinet** (https://energinet.dk), as the Danish TSO, is responsible for the operation of both the power and gas systems at the transmission level.

- In Europe: **ENTSO-E (European Network of Transmission System Operators for Electricity)** coordinates all European TSOs to run the high-voltage grid, set common rules, and coordinate cross-border electricity markets.



ENTSO-e: European Network of Transmission System Operators for Electricity (www.entsoe.eu)

- Distribution System Operator (DSO): Responsible for the safe operation of the underlying power grid at the medium- and low-voltage distribution level (typically a radial grid).
  - A **radial grid** is a **tree-like distribution network** where electricity flows in **one direction** from the substation to consumers, with **no loops**.
  - TSO transmission grids are **meshed** (many loops), whereas distribution grids are **radial** (one path).
  - Examples in Denmark:

    DSOs in Zealand (eastern Denmark)

- Market Regulator: Responsible for monitoring market performance in both the short and long run, as well as designing appropriate market regulations and policies.

  Examples:
  - **The Danish Utility Regulatory** (forsyningstilsynet.dk) in Denmark
  - **Federal Energy Regulatory Commission (FERC)** in the U.S.
  - European Union Agency for the Cooperation of Energy Regulators

- Other market actors:
  - Traders (both physical and purely financial),
  - Balancing Responsible Parties (BRPs),
  - Flexibility Aggregators,

- and other relevant market actors will be introduced throughout the course as needed.

2. Various types of electricity market

Three main markets with different products are:

- Capacity markets
- Energy markets
- Ancillary service markets

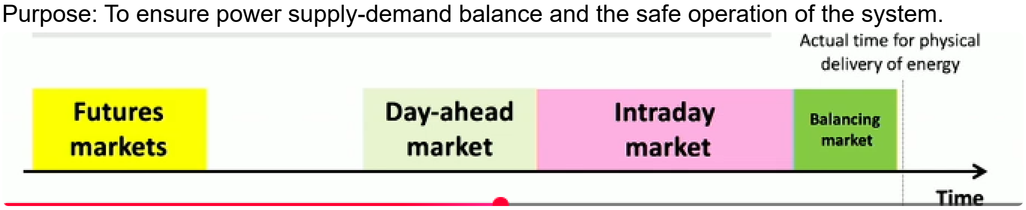
2.1 Capacity markets

- Designed to ensure that **sufficient** generation capacity (measured in MW) is available to maintain supply security and support reliable system operation.
- Any producer submitting an offer to the electricity market—regardless of whether it is accepted or rejected—is eligible for compensation based on their availability (measured in EUR/MW).
- Capacity payments serve as an incentive for power producers to invest in new generation assets over the long term.
- Who pays the bill: The final consumer.
  However, this course will not cover capacity markets.

2.2 Energy market

A central marketplace for exchanging energy (in MWh):

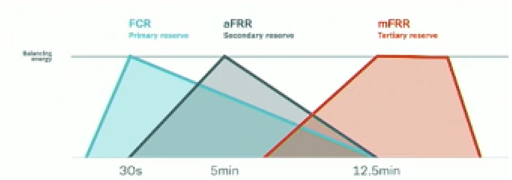
- **1) Futures markets**:
  - Long-term financial contracts (with a time span of up to 6 years)
  - Purpose: Price hedging and risk management for buyers and sellers
  - Example: NASDAQ Commodities in Scandinavian countries
  - Note: This topic will not be covered in this course.
- **2) Day-ahead market**:
  - Also known as the "spot market"
  - Timing: Cleared 12–36 hours before the actual delivery time
  - Example: Nord Pool Elspot, which is cleared at noon on day D-1 for energy exchange during each hour of day D (from midnight to midnight), resulting in different market prices and quantities for each hour.
- **3) Intra-day market**:
  - This market provides an opportunity to "modify" day-ahead schedules in case of updated forecasts, asset failures, or trading purposes. It operates as a continuous and/or discrete auction-based trading platform between the day-ahead market and real-time.
  - Previously it was just a continuous market: A **continuous market** is a trading system where **orders are matched instantly and continuously** whenever a buy and sell order meet—with **no fixed auction time**, trading 24/7, no gate closure, or market clearing.
  - Example: It starts at 3 PM on D-1 and ends 1 hour before real-time. Three discrete auction-based sessions (ID-A) and a continuous platform (ID-C) are available. The current time resolution is 1 hour, but it will shift to 15 minutes soon.
- **4) Balancing market**:
  - Also known as the "imbalance" market or the "real-time" market.
  - This market operates close to real-time system operations.
  - Purpose: To ensure power supply-demand balance and the safe operation of the system.



2.3 Ancillary service markets

These markets allow the system operator to procure services necessary for the secure and reliable operation of the system. E.g.:

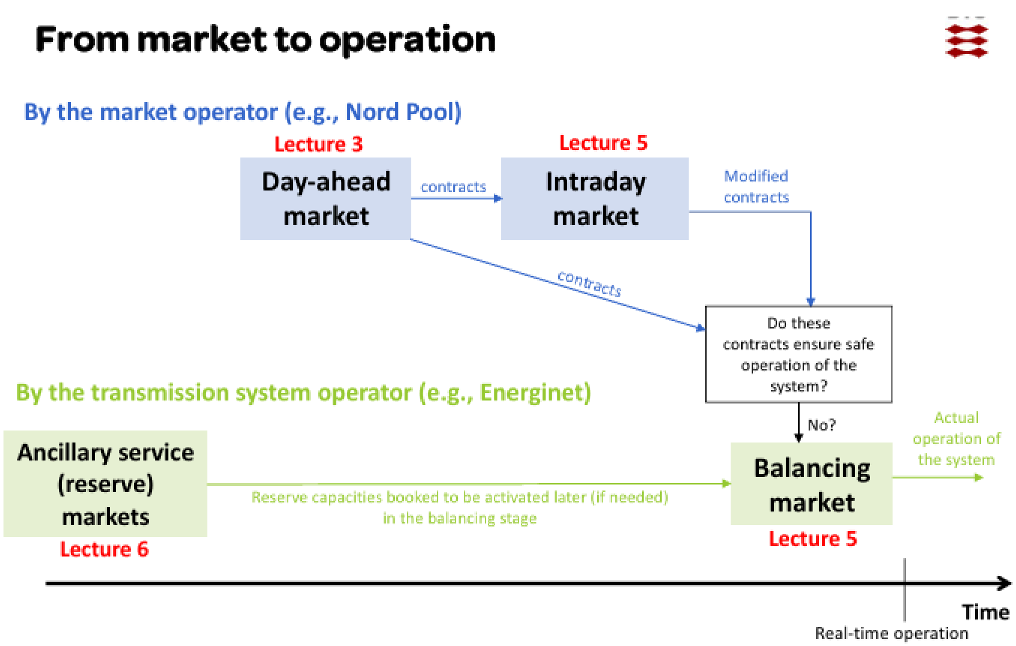
- Primary reserves
- Secondary reserves
- Tertiary reserves
- Black-start capability
- Reactive and voltage-control reserves



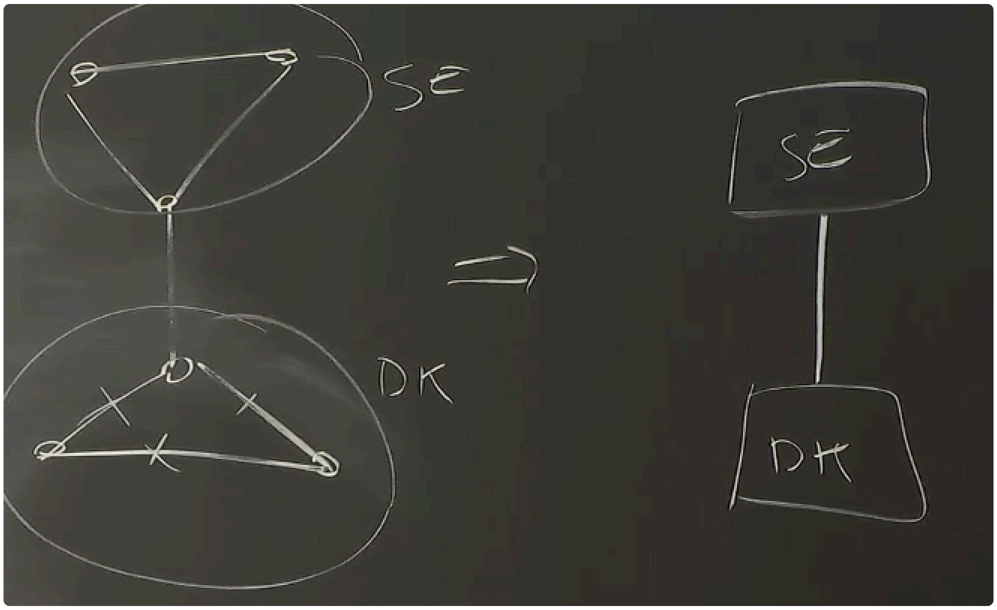
- And more.

> FCR, aFRR, mFRR

- **FCR – Frequency Containment Reserve** *Instant, automatic response* to stabilize frequency after a disturbance (first seconds).
- **aFRR – Automatic Frequency Restoration Reserve** *Automatic ramping* to restore frequency and relieve FCR (seconds → minutes).
- **mFRR – Manual Frequency Restoration Reserve** *Manually activated* reserve (TSO triggers) to restore balance and replace aFRR (minutes → 15 min).



3. European vs U.S.electricity markets



Europe: 6 nodes (for example) -> split 2 zones

In terms of market and system operation:

- In Europe: Market operators and system operators are separate entities (e.g., Nord Pool as the market operator and Energinet as the TSO).
- In the U.S.: An Independent System Operator (ISO) is responsible for both clearing the market and operating the system (e.g., CAISO and PJM).

In terms of network modeling for market clearing:

- In Europe: The transmission network is modeled in a simplified manner (using zonal representation) within the day-ahead and intraday market-clearing process. There is no detailed network modeling within each bidding zone.
- In the U.S.: The transmission network is fully modeled using a nodal representation and a linearized power flow model in all market-clearing problems.

In terms of energy and reserve markets:

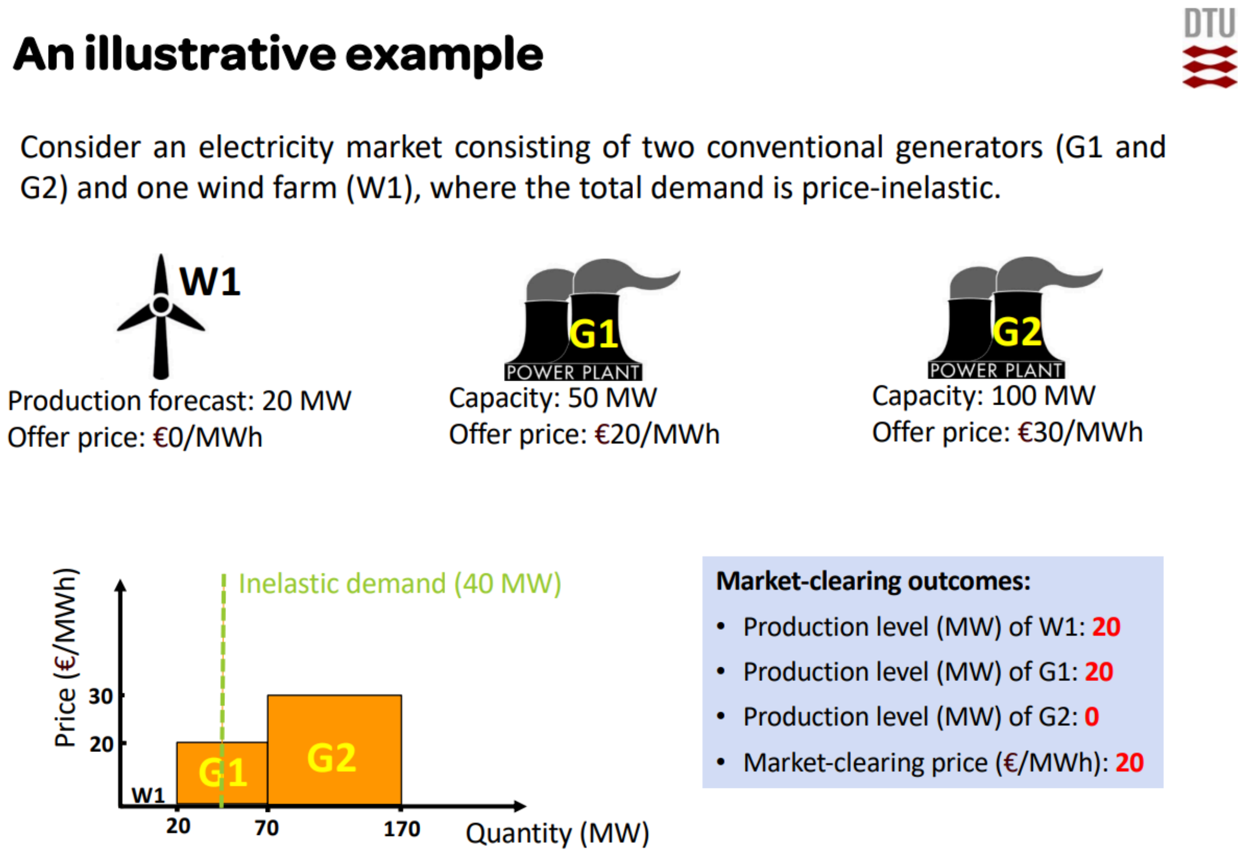
- In Europe: The reserve markets are cleared separately by the TSOs (e.g., Energinet) before the day-ahead energy market is cleared by the market operator.
- In the U.S.: A joint energy and reserve market exists in the day-ahead time stage, cleared by the ISO, resulting in an energy and reserve dispatch co-optimization problem.

European vs U.S. electricity markets
Regarding complex offers versus complex market clearing:

- **In Europe:** Market actors, such as power producers, are responsible for ensuring their offers/bids adhere to their technical constraints (e.g., ramp limits, minimum production limits, and minimum up/down time limits for conventional generators). Market actors internalize their technical limits within their own offers/bids: They offer/bid for their entire portfolio within a bidding zone (not per asset). This results in complex orders (bids and offers), while the market-clearing process remains relatively simple.

3. Market-clearing as a linear optimization problem

An illustrative problem



Inelastic demand: Consumers buy the same amount no matter the price.
-> the price does NOT affect demand - demand is fixed

The market-clearing price is determined **only by generator offers**, because demand does **not change** when price changes.

Example: Demand = 40 MW (inelastic) Even if price is €10, €20, or €100 → demand stays **40 MW**.

Primal variables:

$$p_{W1},\ p_{G1},\ p_{G2}$$

$$\min\ C = 0 \cdot p_{W1} + 20 \cdot p_{G1} + 30 \cdot p_{G2}$$

$$0 \le p_{W1} \le 20$$

$$0 \le p_{G1} \le 50$$

$$0 \le p_{G2} \le 100$$

Power balance (inelastic demand):

$$p_{W1} + p_{G1} + p_{G2} = D$$

Dual variables:

$$\lambda = \text{market-clearing price}$$

Numerical example ($D = 40$ MW):

$$p_{W1} = 20, \quad p_{G1} = 20, \quad p_{G2} = 0$$

Market-clearing price:

$$\lambda = 20\ €/\mathrm{MWh}$$

A **dual variable** is the **shadow price** of a constraint — it tells you **how much the objective would improve if you relax that constraint by 1 unit**.

In electricity markets:

- The **dual variable of the power-balance constraint** = **market-clearing price**
- It represents the **value of one extra MW of demand**
- If demand increases by 1 MW, the objective cost increases by λ (the dual)

Example constraint:

$$p_{W1} + p_{G1} + p_{G2} = D$$

Dual variable:

$$\lambda = 20\,€/\mathrm{MWh}$$

Meaning: If demand increases by **1 MW**, total system cost increases by **€20** → that's the **market price**.

μ: The dual variable for an inequality constraint. It measures the **shadow price** of relaxing that constraint by **1 unit**.

μ **is the dual variable for an inequality constraint.** It tells you whether a limit is **active (binding)**.

- If the limit **binds** → $\mu > 0$
- If the limit **does NOT bind** → $\mu = 0$

Example constraint:

$$p_{G1} \leq 50$$

- If $p_{G1} = 50$ → limit is active → $\mu > 0$
- If $p_{G1} \ \ 50$ → limit is not active → $\mu = 0$

In your numerical example, G1 produces **20**, so the limit is not binding:

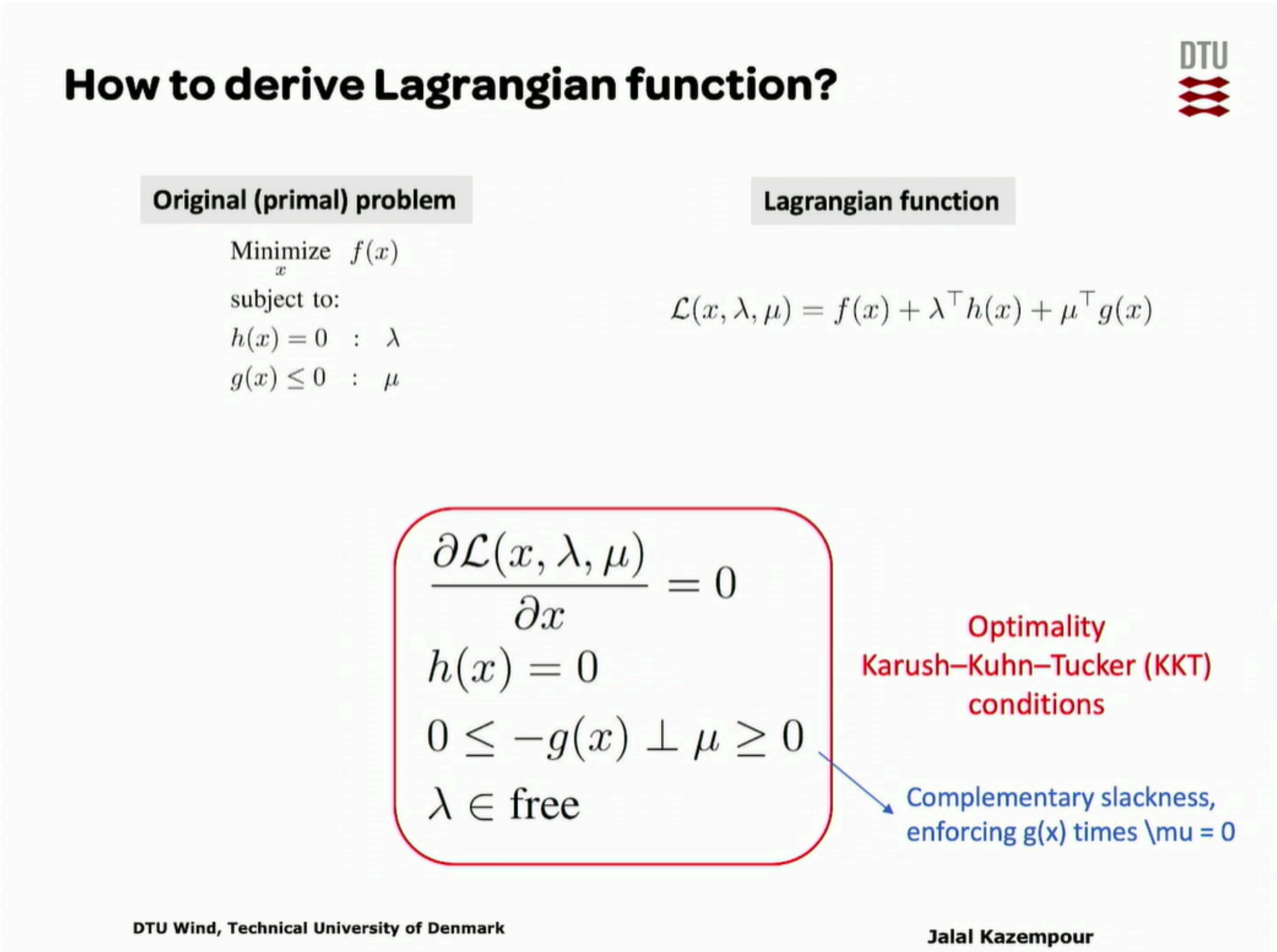
$$\mu = 0$$

The **Lagrange equation** (Lagrangian) is what you get when you combine:

- the **objective function**, and
- all the **constraints**,
- multiplied by their **dual variables** (λ, μ).

Goal: To combine the **objective** and **all constraints** into **one single function** so we can apply the **KKT conditions** and find the **optimal solution + optimal prices (dual variables)**.

KKT: Karush-Kuhn-Tucker conditions



Objective:

$$\min C = 0p_{W1} + 20p_{G1} + 30p_{G2}$$

Constraints:

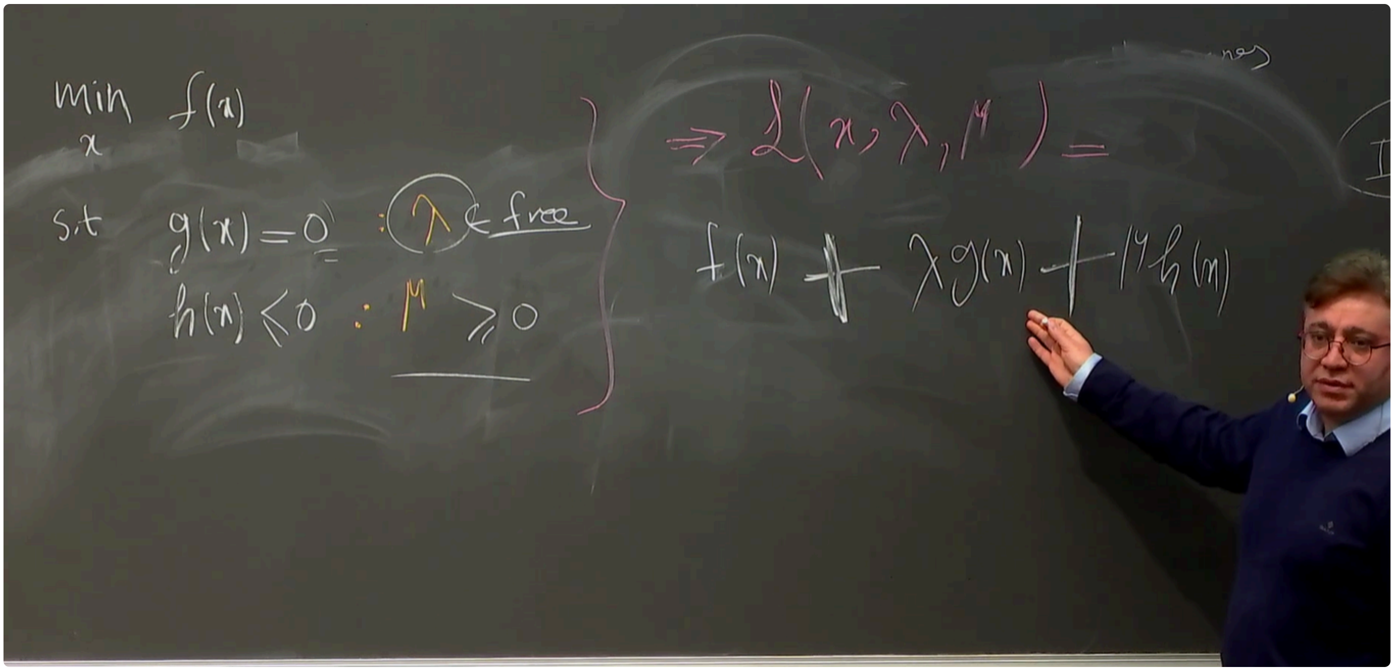
$$p_{W1} + p_{G1} + p_{G2} = D \quad (\lambda)$$
$$0 \leq p_i \leq p_i \quad (\mu_i,\ \mu_{upi})$$

Lagrangian Function

$$= 0p_{W1} + 20p_{G1} + 30p_{G2} + \lambda(D \;\; p_{W1} \;\; p_{G1} \;\; p_{G2}) + \mu_{upW1}(p_{W1} \;\; 20) + \mu_{upG1}(p_{G1} \;\; 50) + \mu_{upG2}(p_{G2} \;\; 100)$$

(Note: Usually, bounds are written as $p_i \;\; p_{upi} \leq 0$, so we add $\mu_{upi}(p_i \;\; p_{upi})$).

That’s the **Lagrange equation**: **objective + duals × constraints**.



h(x)

- These are **equality constraints.**
- They must hold **exactly.**
  Example in your market-clearing probl

$$h(x) = p_{W1} + p_{G1} + p_{G2} - D = 0$$

g(x)

- These are **inequality constraints.**
- They must satisfy **≤ 0.**
  Example for generator limits:

$$g(x) = p_{G1} - 50 \le 0$$

μ

μ is the dual variable for the *upper bound* of an inequality constraint.

- Every variable has **two limits**:
  - **lower limit** → μ (or $\underline{\mu}$)
  - **upper limit** → μ^{up} (or $\overline{\mu}$)
    So:
- If the **upper limit is binding** → $\overline{\mu} > 0$
- If the **upper limit is NOT binding** → $\overline{\mu} = 0$

Optimization problem for market clearing

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Our optimization problem

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\,p^{\mathrm{W1}} + 20\,p^{\mathrm{G1}} + 30\,p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Rewritten to a standard form

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\,p^{\mathrm{W1}} + 20\,p^{\mathrm{G1}} + 30\,p^{\mathrm{G2}}$$

subject to:

$$-p^{\mathrm{W1}} \leq 0 \quad : \quad \underline{\mu}^{\mathrm{W1}}$$

$$p^{\mathrm{W1}} - 20 \leq 0 \quad : \quad \overline{\mu}^{\mathrm{W1}}$$

$$-p^{\mathrm{G1}} \leq 0 \quad : \quad \underline{\mu}^{\mathrm{G1}}$$

$$p^{\mathrm{G1}} - 50 \leq 0 \quad : \quad \overline{\mu}^{\mathrm{G1}}$$

$$-p^{\mathrm{G2}} \leq 0 \quad : \quad \underline{\mu}^{\mathrm{G2}}$$

$$p^{\mathrm{G2}} - 100 \leq 0 \quad : \quad \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Lagrangian function of the market-clearing optimization problem

Standard form of our optimization problem

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$\begin{aligned}
&-p^{\mathrm{W1}} \leq 0 \ : \ \underline{\mu}^{\mathrm{W1}}\\
&p^{\mathrm{W1}} - 20 \leq 0 \ : \ \overline{\mu}^{\mathrm{W1}}\\
&-p^{\mathrm{G1}} \leq 0 \ : \ \underline{\mu}^{\mathrm{G1}}\\
&p^{\mathrm{G1}} - 50 \leq 0 \ : \ \overline{\mu}^{\mathrm{G1}}\\
&-p^{\mathrm{G2}} \leq 0 \ : \ \underline{\mu}^{\mathrm{G2}}\\
&p^{\mathrm{G2}} - 100 \leq 0 \ : \ \overline{\mu}^{\mathrm{G2}}\\
&40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \ : \ \lambda
\end{aligned}$$

Lagrangian function

$$\begin{aligned}
\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) = \\
& 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}\\
& - p^{\mathrm{W1}}\underline{\mu}^{\mathrm{W1}} + \left(p^{\mathrm{W1}} - 20\right)\overline{\mu}^{\mathrm{W1}}\\
& - p^{\mathrm{G1}}\underline{\mu}^{\mathrm{G1}} + \left(p^{\mathrm{G1}} - 50\right)\overline{\mu}^{\mathrm{G1}}\\
& - p^{\mathrm{G2}}\underline{\mu}^{\mathrm{G2}} + \left(p^{\mathrm{G2}} - 100\right)\overline{\mu}^{\mathrm{G2}}\\
& + \lambda\left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)
\end{aligned}$$

KKT Conditions of the market-clearing optimization problem:

KKT conditions of the market-clearing optimization problem

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$$\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) =$$

$$\begin{aligned}
&0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\
&- p^{\mathrm{W1}}\underline{\mu}^{\mathrm{W1}} + \left(p^{\mathrm{W1}} - 20\right)\overline{\mu}^{\mathrm{W1}} \\
&- p^{\mathrm{G1}}\underline{\mu}^{\mathrm{G1}} + \left(p^{\mathrm{G1}} - 50\right)\overline{\mu}^{\mathrm{G1}} \\
&- p^{\mathrm{G2}}\underline{\mu}^{\mathrm{G2}} + \left(p^{\mathrm{G2}} - 100\right)\overline{\mu}^{\mathrm{G2}} \\
&+ \lambda\left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)
\end{aligned}$$

KKT conditions

$$\boxed{\begin{aligned}
&\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{W1}}} = 0 + \overline{\mu}^{\mathrm{W1}} - \underline{\mu}^{\mathrm{W1}} - \lambda = 0 && 0 \le \left(20 - p^{\mathrm{W1}}\right) \perp \overline{\mu}^{\mathrm{W1}} \ge 0 \\
& && 0 \le p^{\mathrm{W1}} \perp \underline{\mu}^{\mathrm{W1}} \ge 0 \\
&\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G1}}} = 20 + \overline{\mu}^{\mathrm{G1}} - \underline{\mu}^{\mathrm{G1}} - \lambda = 0 && 0 \le \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \ge 0 \\
& && 0 \le p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \ge 0 \\
&\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G2}}} = 30 + \overline{\mu}^{\mathrm{G2}} - \underline{\mu}^{\mathrm{G2}} - \lambda = 0 && 0 \le \left(100 - p^{\mathrm{G2}}\right) \perp \overline{\mu}^{\mathrm{G2}} \ge 0 \\
&40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 && 0 \le p^{\mathrm{G2}} \perp \underline{\mu}^{\mathrm{G2}} \ge 0
\end{aligned}}$$

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KTT Conditions of the market-clearing optimisation

KKT conditions of the market-clearing optimization problem

$$\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) =$$

$$\begin{aligned} & 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\ & - p^{\mathrm{W1}} \underline{\mu}^{\mathrm{W1}} + (p^{\mathrm{W1}} - 20)\, \overline{\mu}^{\mathrm{W1}} \\ & - p^{\mathrm{G1}} \underline{\mu}^{\mathrm{G1}} + (p^{\mathrm{G1}} - 50)\, \overline{\mu}^{\mathrm{G1}} \\ & - p^{\mathrm{G2}} \underline{\mu}^{\mathrm{G2}} + (p^{\mathrm{G2}} - 100)\, \overline{\mu}^{\mathrm{G2}} \\ & + \lambda \left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right) \end{aligned}$$

KKT conditions

We can solve KKT conditions as a system of equations!

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{W1}}} = 0 + \overline{\mu}^{\mathrm{W1}} - \underline{\mu}^{\mathrm{W1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G1}}} = 20 + \overline{\mu}^{\mathrm{G1}} - \underline{\mu}^{\mathrm{G1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G2}}} = 30 + \overline{\mu}^{\mathrm{G2}} - \underline{\mu}^{\mathrm{G2}} - \lambda = 0$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0$$

$$0 \le \left(20 - p^{\mathrm{W1}}\right) \perp \overline{\mu}^{\mathrm{W1}} \ge 0$$

$$0 \le p^{\mathrm{W1}} \perp \underline{\mu}^{\mathrm{W1}} \ge 0$$

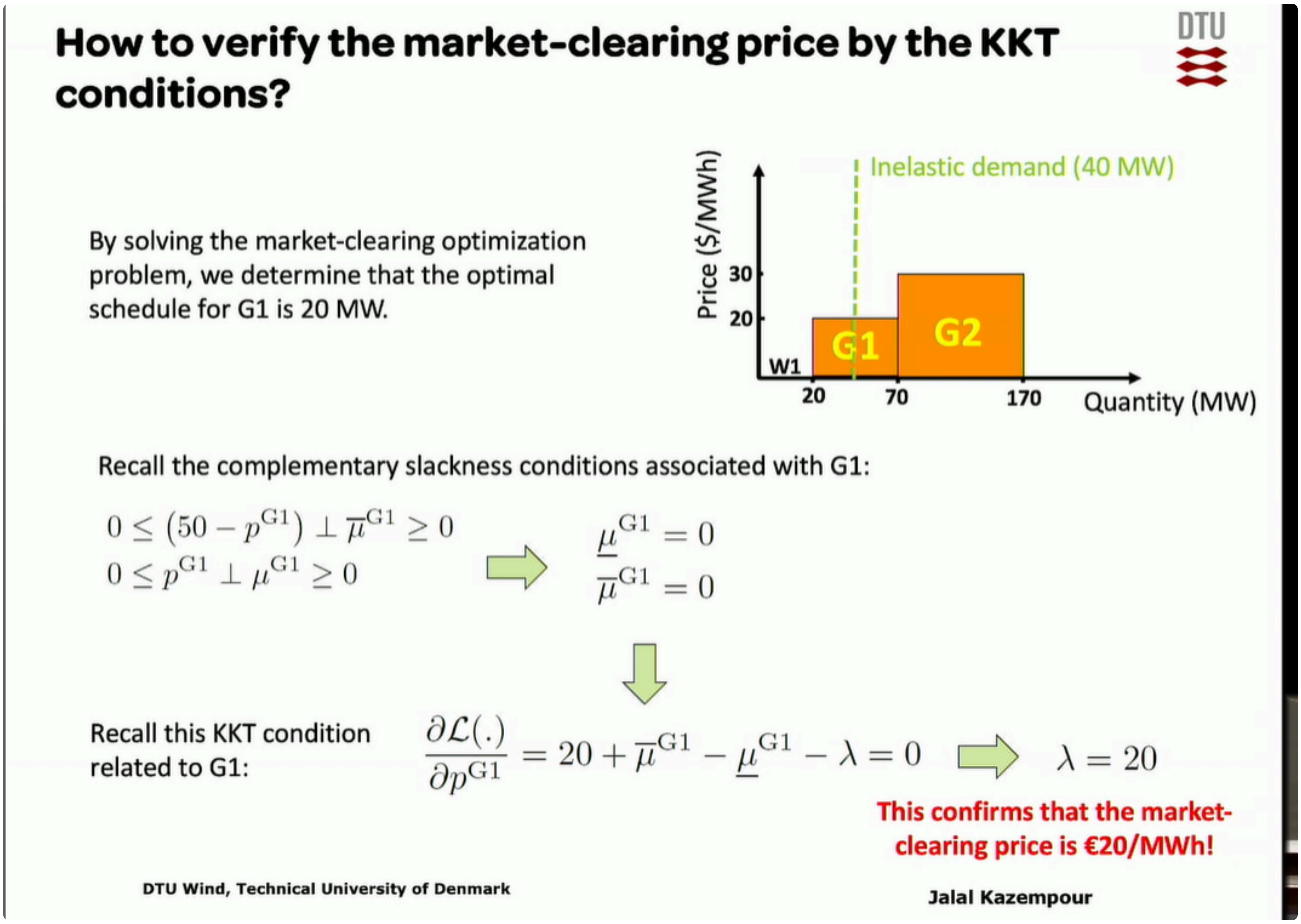
$$0 \le \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \ge 0$$

$$0 \le p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \ge 0$$

$$0 \le \left(100 - p^{\mathrm{G2}}\right) \perp \overline{\mu}^{\mathrm{G2}} \ge 0$$

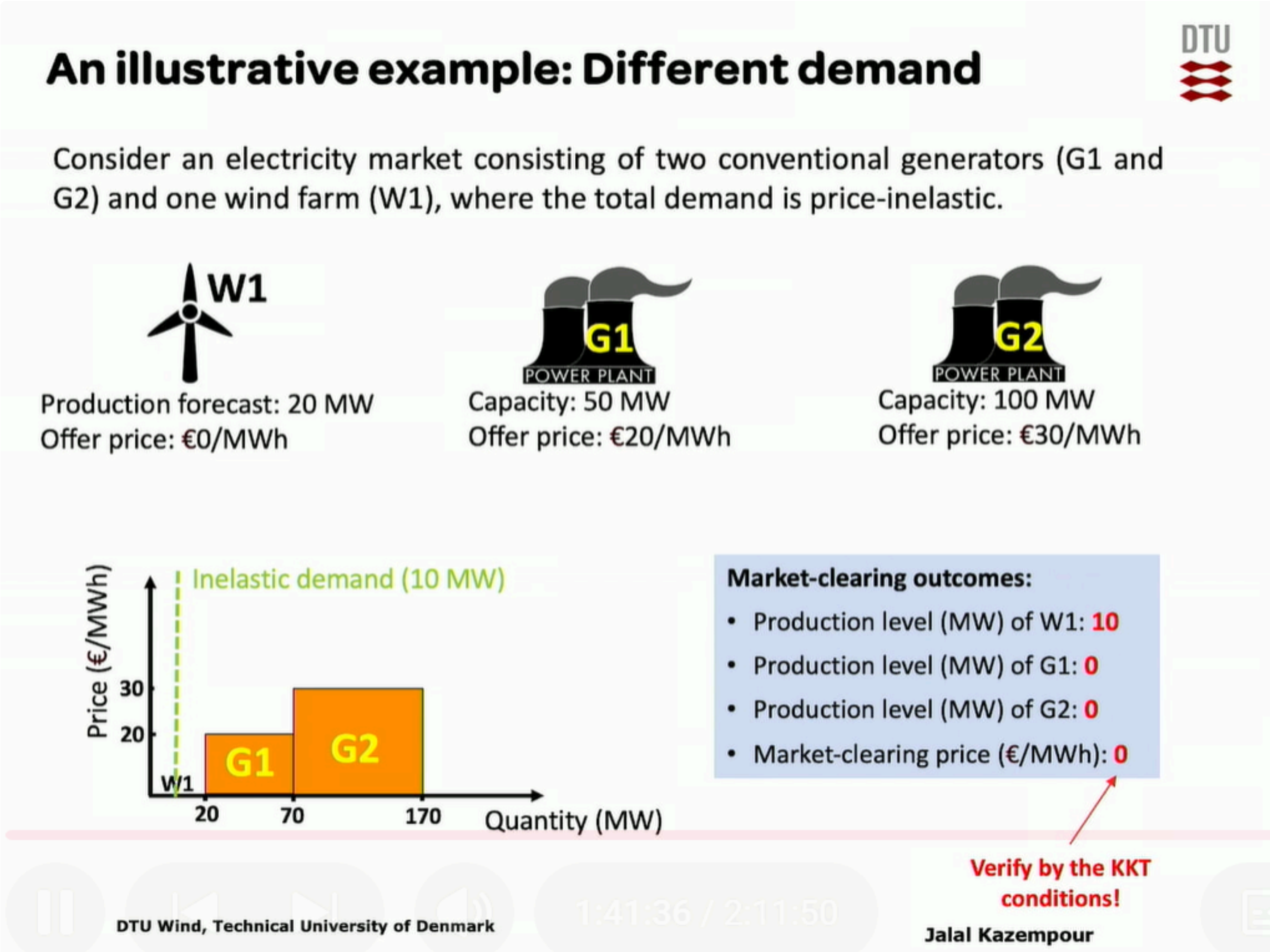
$$0 \le p^{\mathrm{G2}} \perp \underline{\mu}^{\mathrm{G2}} \ge 0$$

How to verify the market-clearing price by the KKT conditions

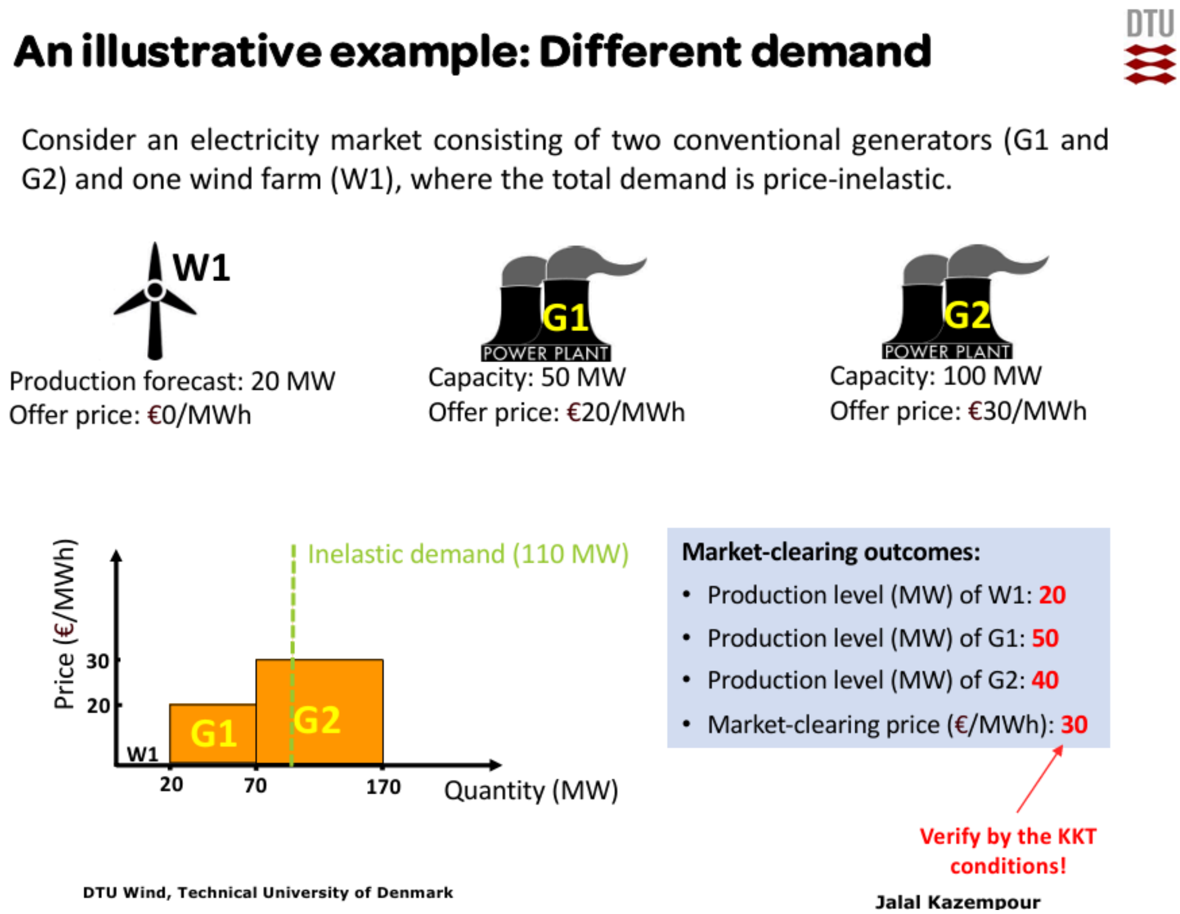


Example with different demand

First, if the demand is 0

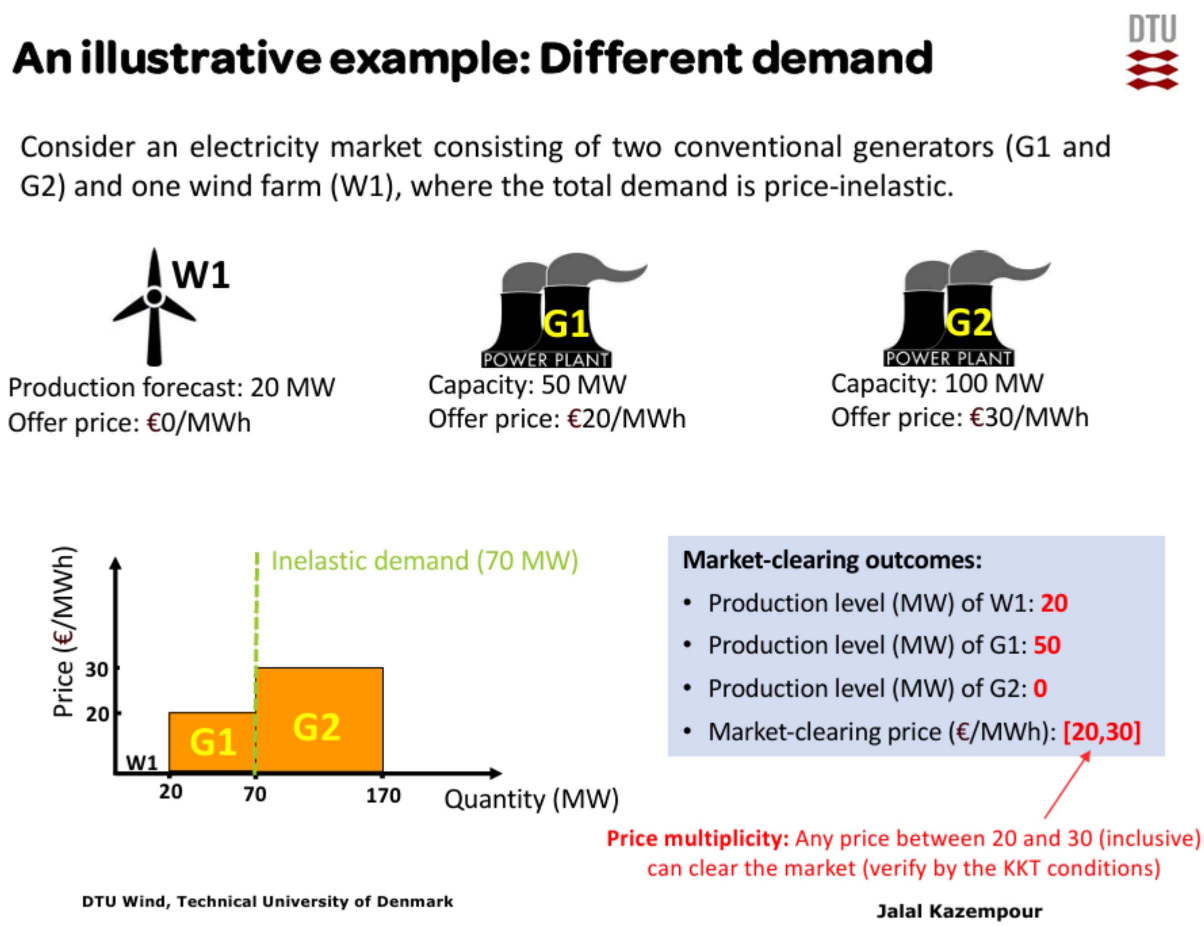


What is the price in here, if the demand quantity is around 80 MW



What is the price in here, if the demand is 70

The right and left hand side if we decrease and increase the demand a bit -> the price will vary -> therefore, it will be in the range of [20,30]



An Illustrative example: Price-elastic demand

Now the demand will be affected by the price

An illustrative example: Price-elastic demand

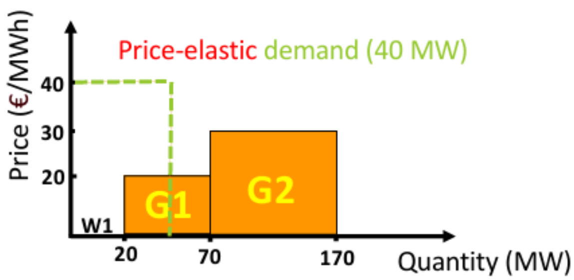
Consider an electricity market consisting of two conventional generators (G1 and G2) and one wind farm (W1). The single demand (D1) is elastic to price.

W1
Production forecast: 20 MW
Offer price: €0/MWh

G1 POWER PLANT
Capacity: 50 MW
Offer price: €20/MWh

G2 POWER PLANT
Capacity: 100 MW
Offer price: €30/MWh

Demand D1
Maximum load: 40 MW
Bid price: €40/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **20**
- Production level (MW) of G2: **0**
- Consumption level (MW) of D1: **40**
- Market-clearing price (€/MWh): **20**

Verify by the KKT conditions!

Market clearing with price-elastic demand

Market clearing with price-elastic demand

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$$\underset{p^{\mathrm{D1}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Maximize}} \quad \underbrace{40\ p^{\mathrm{D1}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}}_{\text{Objective function: Social welfare}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

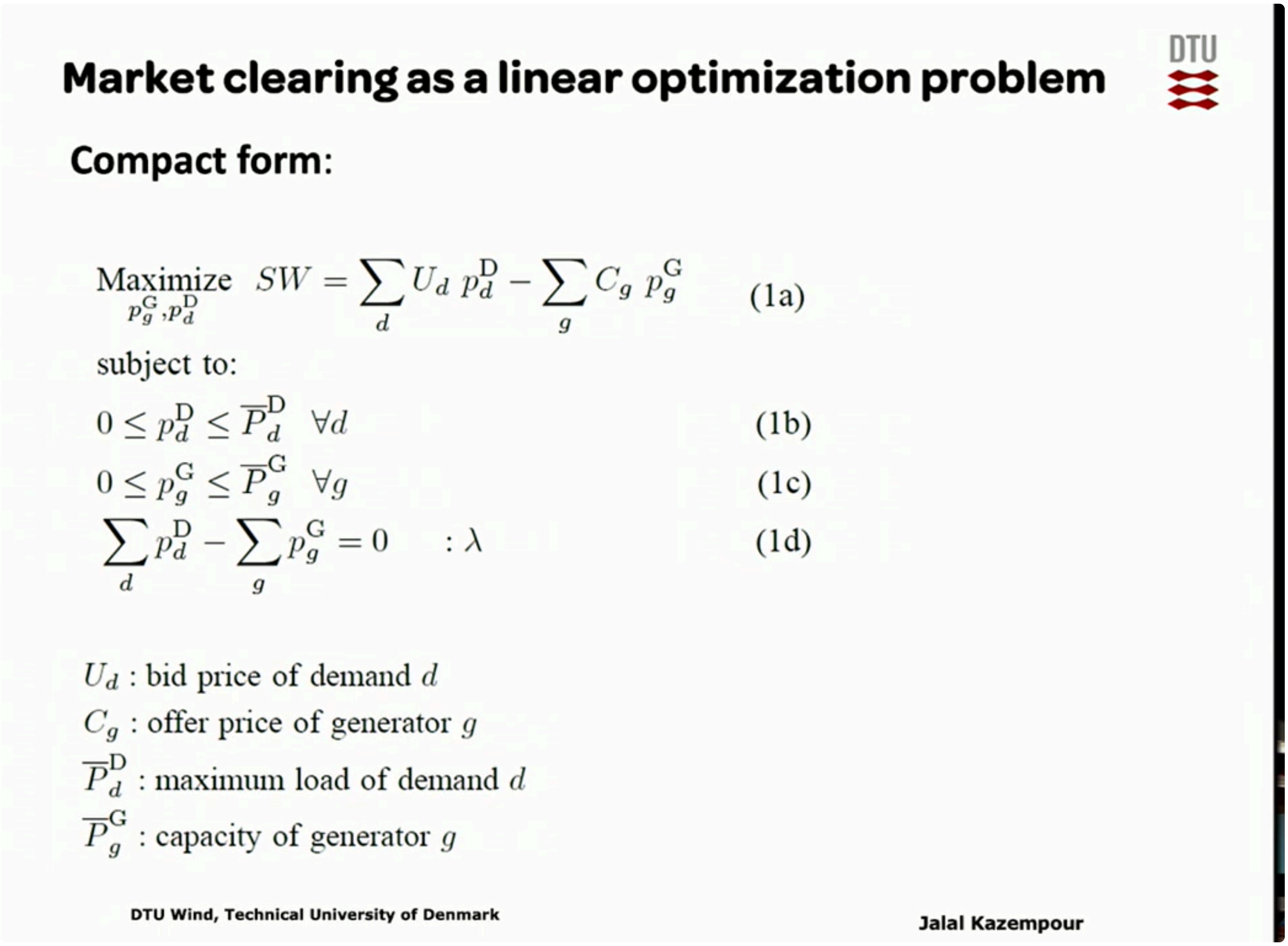
$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$0 \leq p^{\mathrm{D1}} \leq 40 \quad : \quad \underline{\mu}^{\mathrm{D1}}, \overline{\mu}^{\mathrm{D1}}$$

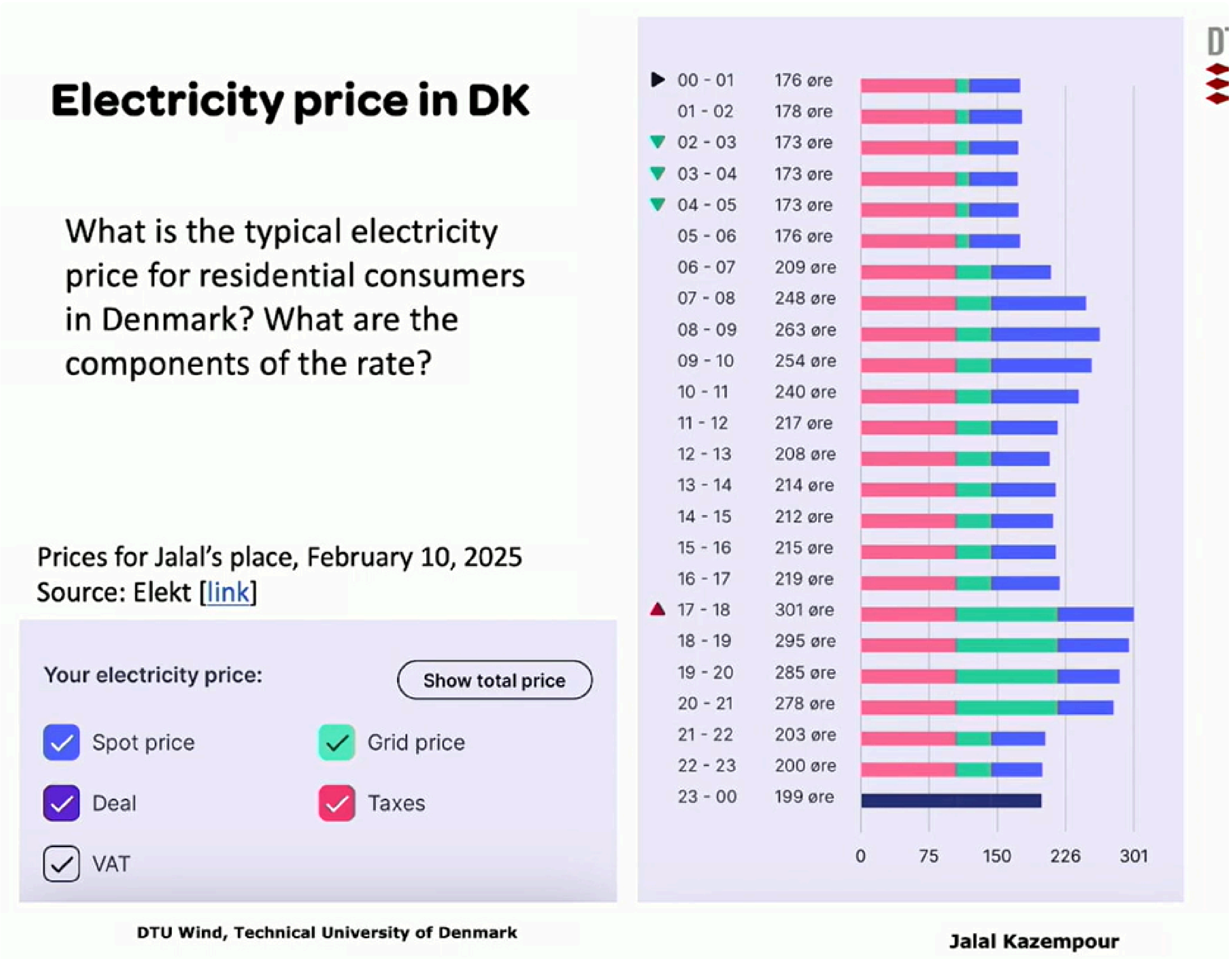
$$p^{\mathrm{D1}} - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Market clearing as a linear optimisation problem



SW: Social welfare

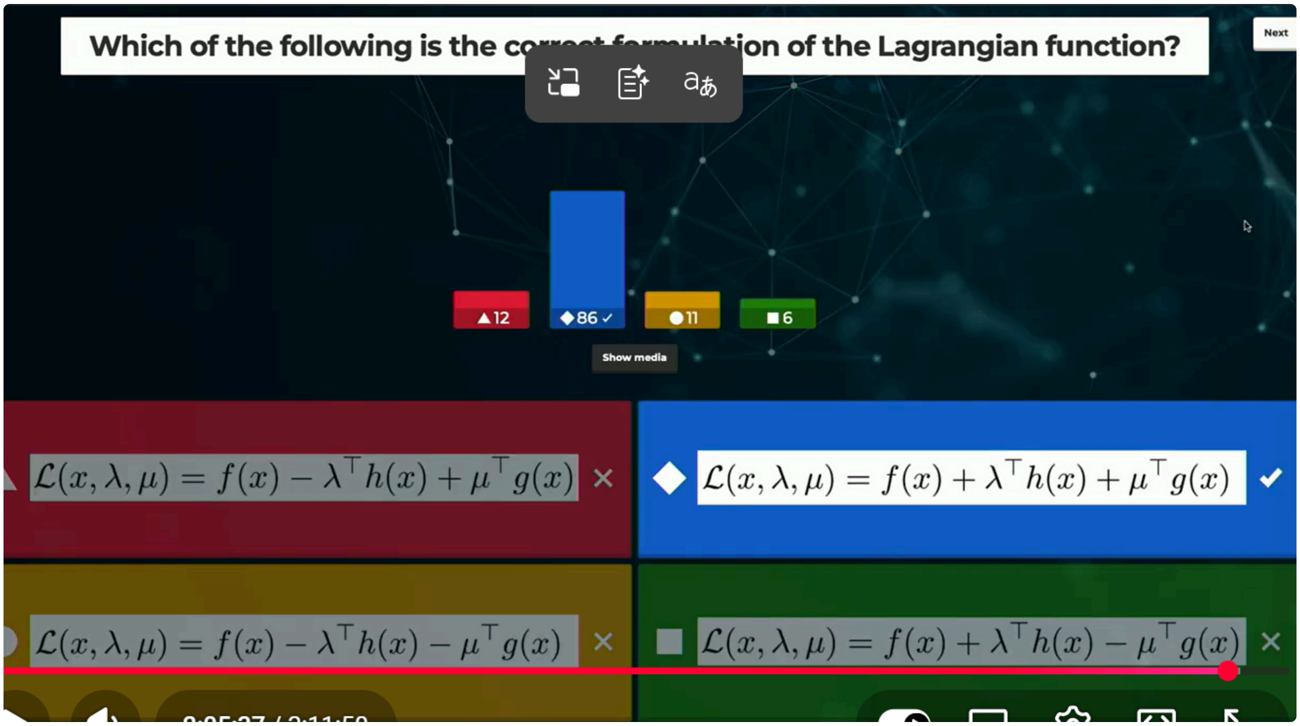
Electricity price in DK

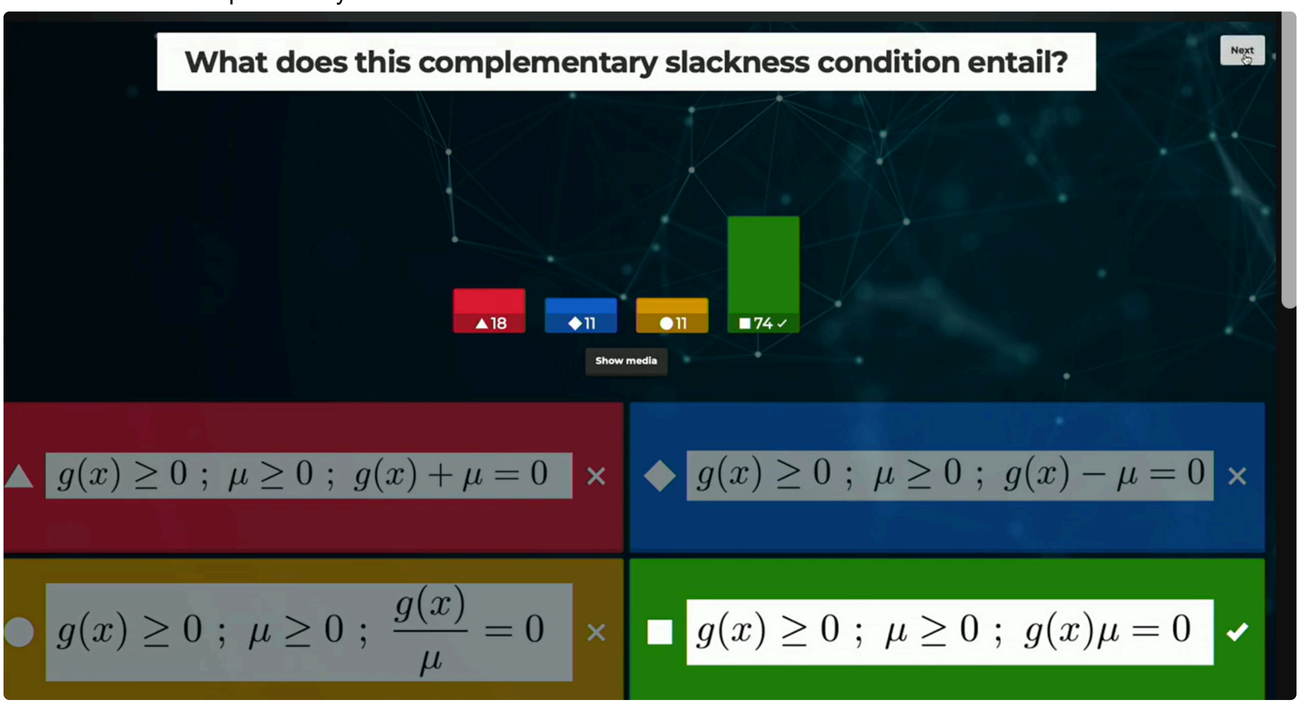


Kahoot

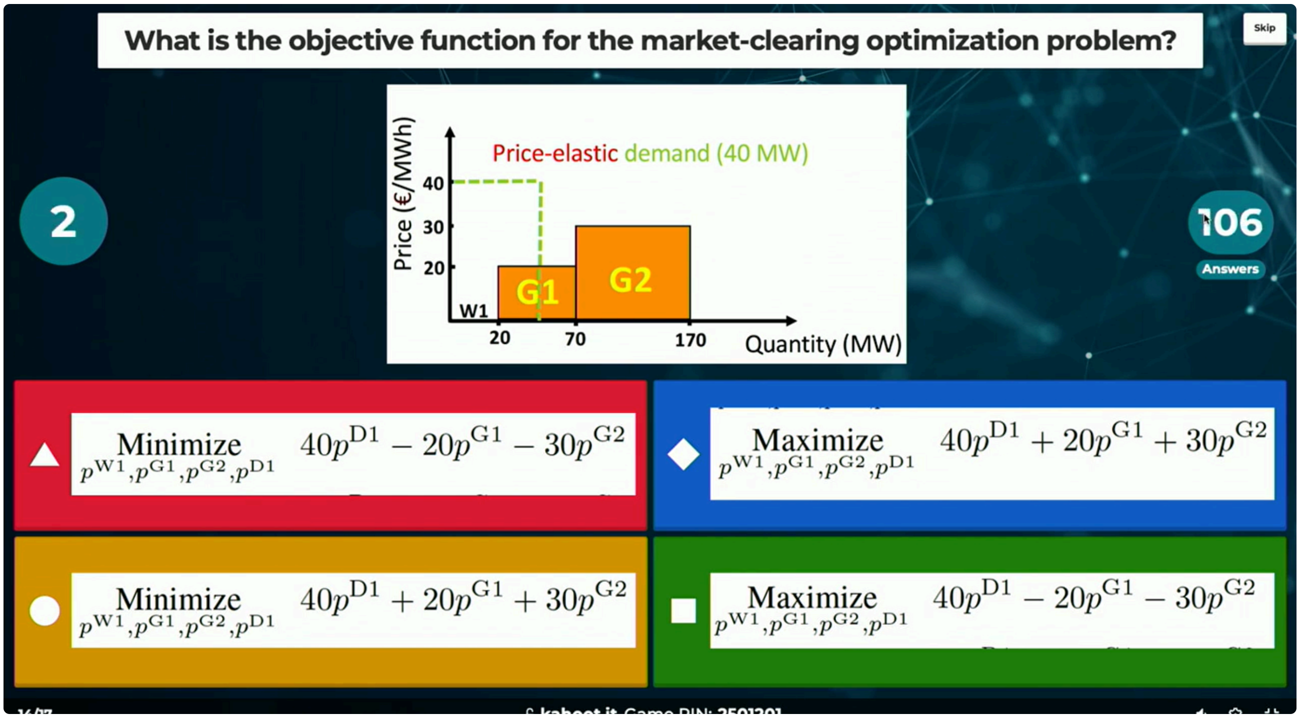
- Q1: A retailer: Buys power from the market in bulk and sells it to small consumers.
- Q2: Which of following choices include only market operators?
  - Nord Pool, EPEX, EEX (a leading international commodity exchange based in Leipizig, Germany)
  - EPEX: European Power Exchange (a major pan-European marketplace where short-term electricity contracts are brought and sold).
- Q3: A transmission system operator (TSO): operates the high-voltage transmisison grid.
- Q4: The Nordic TSOs are Energinet, Svenska Krafnat, Stattnet, Fingrid
- Q5: Nord Pool clears Day a head, intraday
  - It doesn't reserve, and balancing markets
    - In Denmark:
    - **Energinet** clears:
      - FCR (primary reserves)
      - aFRR (secondary reserves)
      - mFRR (manual reserves)
      - Balancing market (real-time)
        In Finland:
    - **Fingrid** clears the same types of reserve and balancing markets.
- Q6: Which entities coordinate national TSOs and regulators, respectively, at the European Level?
  - ENTSO-e and ACER
- Q7: A residental customer at a certain addres can choose her [...] among various options

- Retailer: A residential customer **can choose her electricity retailer** because retailers are **competitive companies**. They offer different:
    - prices (fixed/variable)
    - contract types
    - green energy options
      Retailers buy electricity from markets (day-ahead, intraday) and sell it to households. So customers are free to switch between them
    - **2. TSO/DSO = no choice**
      A customer **cannot choose** the **TSO** or **DSO** because:
  - They are **natural monopolies** tied to the physical grid.
  - The **TSO** runs the national transmission grid (e.g., Energinet, Fingrid).
  - The **DSO** runs the local distribution grid (e.g., Radius, Cerius).
  - They are **regulated**, not competitive.
  - They **do not sell electricity** — they only transport it and ensure reliability.
- Q8: Which choice correctly represents the chronological order of market clearing?
  - Future, day-ahead, intraday, balancing
- Q9: How many zone exists in the following countries, respectively: Denmark, Norway, Sweden, Finland, Germany and France?
  - 2, 5, 4, 1,1,1
- Q10: Which of the following is correct about European electricity markets?
  - The network is simplistically represented in a zone setup.
- Q11: Which of the following is NOT correct about US electricity market?
  - ISO clears the reserve market after the energy market
  - Correct:
    - **1. ISO = both market operator AND system operator**
      U.S. ISOs/RTOs (e.g., CAISO, PJM) **clear the market AND operate the grid**. This is different from Europe, where Nord Pool ≠ Energinet.
  - **2. Network is fully modeled (nodal)**
    - U.S. markets use **nodal pricing** with a **linearized power-flow model** in all clearing problems.
  - **3. Joint energy + reserve co-optimization**
    In the U.S., **day-ahead energy and reserve markets are cleared together** by the ISO.
  - **4. Simple bids, complex clearing**:
    - Producers submit **technical constraints** (ramp limits, min up/down times, etc.). This makes the clearing problem a **unit commitment** with binary variables.
  - NOT Correct: In the **U.S. ISO/RTO markets**, the **day-ahead energy market and reserve market are cleared *together*** in **one single co-optimization**. From here
    - **Nodal model**
    - **Joint clearing of energy + reserves**
    - **Unit commitment with binary variables**
- Q12: Which of the following is the correct formulation of the Lagrangian function?

- 

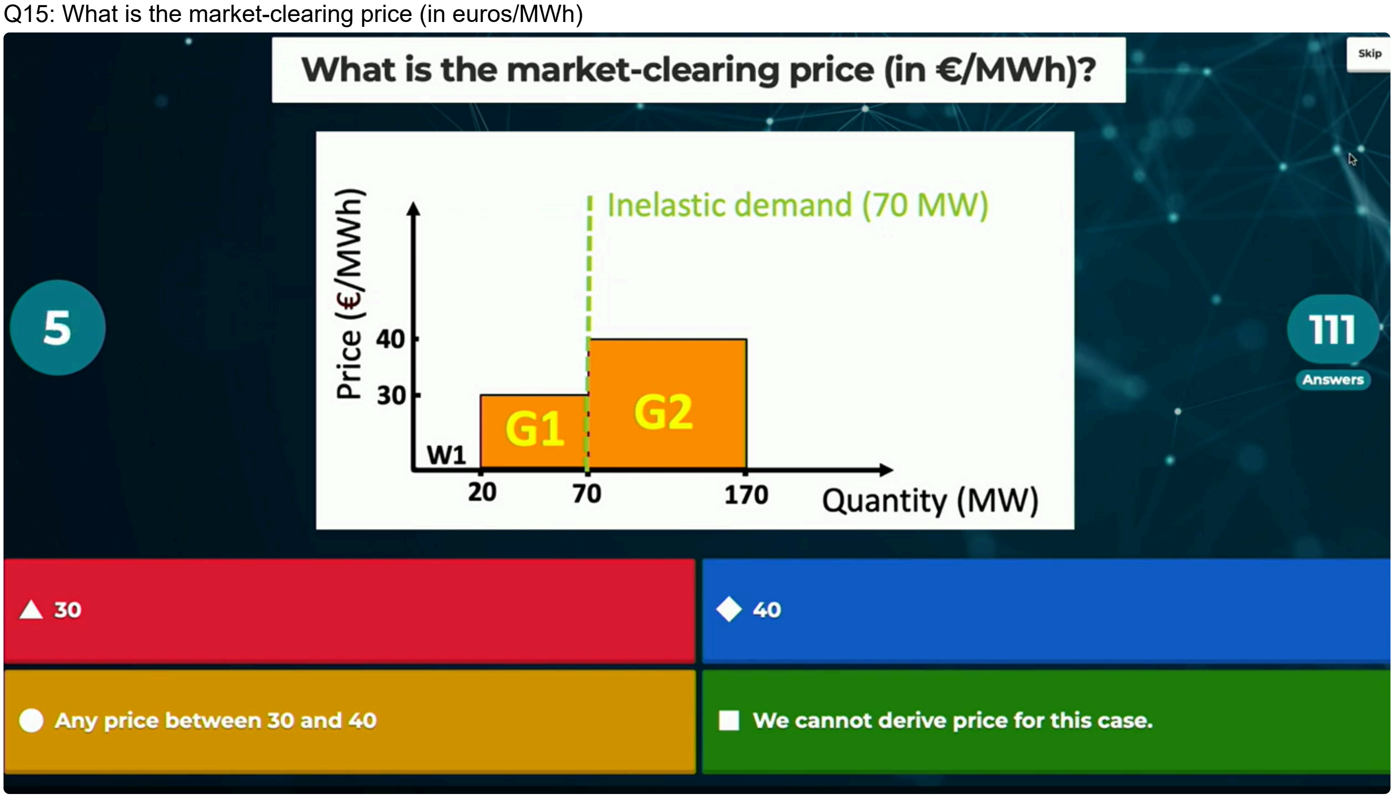
- Q13: Which does this complementary slackness condition entail?
  - 

Q14: What is the objective function for the market-clearing optimisation problem?



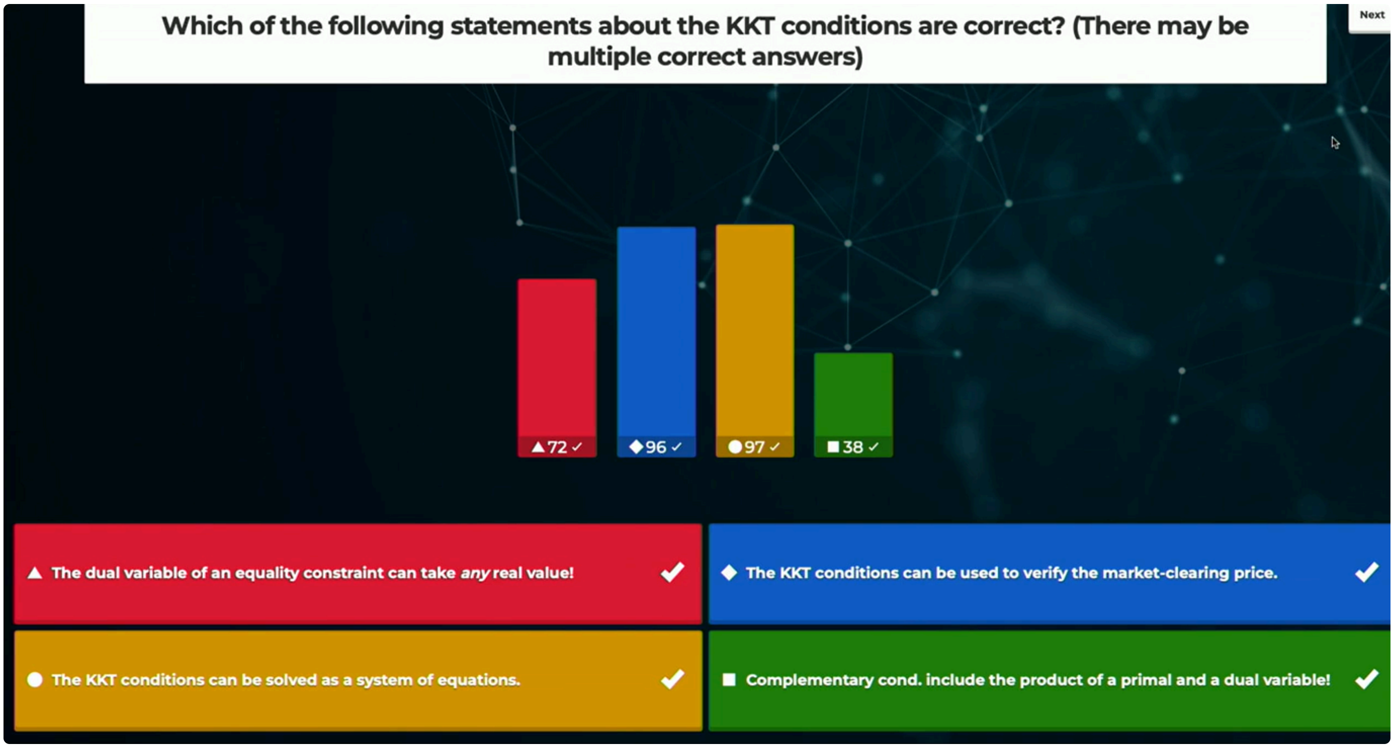
Correct answer: D - Elastic Demand -M maximise the objective function

Q15: What is the market-clearing price (in euros/MWh)



Any price between 30 and 40

Q16: Which of the following statements about the KKT conditions are correct (there maybe multiple correct answers)



- **1. The dual variable of an equality constraint can take any real value**

  ✅ Correct

  - Equality constraints h(x)=0 have **no sign restriction** on their dual variable λ.
  - It can be **positive, negative, or zero**, depending on whether relaxing the constraint increases or decreases the objective.
  - In contrast, inequality constraints g(x)≤0 have **non-negative** duals μ≥0.
- **2. The KKT conditions can be used to verify the market-clearing price*

  ✅ Correct

  - In electricity markets, the **dual variable of the power-balance constraint** equals the **market-clearing price**.
  - Solving the KKT system gives both **optimal dispatch (primal)** and **prices (dual)**.
  - Hence, checking the KKT conditions confirms that the computed price satisfies optimality.
- **3. The KKT conditions can be solved as a system of equations**

  ✅ Correct

  - The KKT conditions combine
    - **Stationarity** (gradient equations),
    - **Primal feasibility** (constraints),
    - **Dual feasibility** (sign restrictions),
    - **Complementary slackness** (products = 0).
  - Together, they form a **system of equations and inequalities** that can be solved numerically.

    4. Complementary condition includes the product of a primal and a dual variable

    ✅ Correct

  - For each inequality constraint gi(x)≤0, the KKT condition requires:

$$\mu i \cdot gi(x) = 0$$

  - This means either the constraint is **active** (gi(x)=0,μi>0) or **inactive** (gi(x)<0,μi=0).
  - That product term is the **complementary slackness condition**.
- Q17: What is the common range of electricity prices typial residental consumers in Denmark (in Euro/KWh, including Tax)?

- 0.2 - 0.5