46755 – Renewables in Electricity Markets

Lecture 2: Fundamentals of electricity markets

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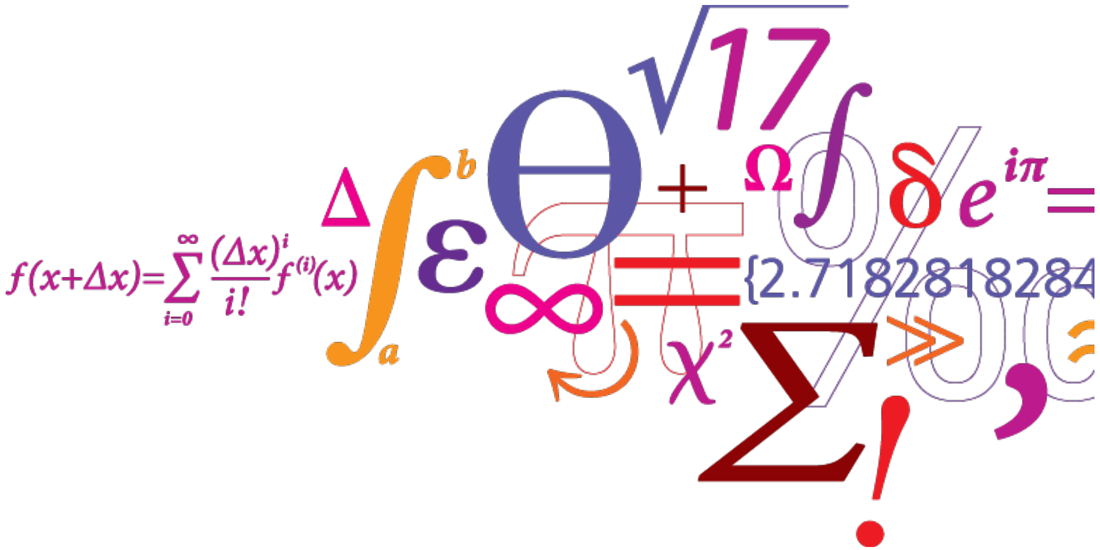
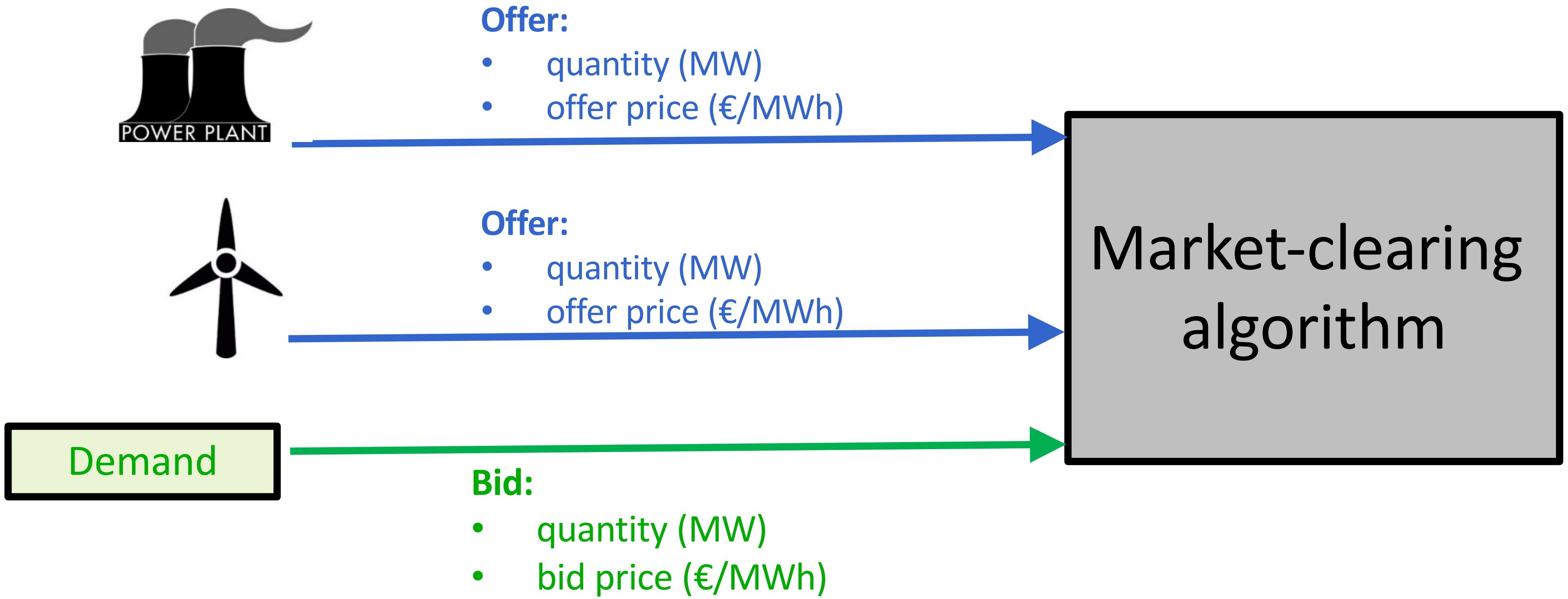


Recap

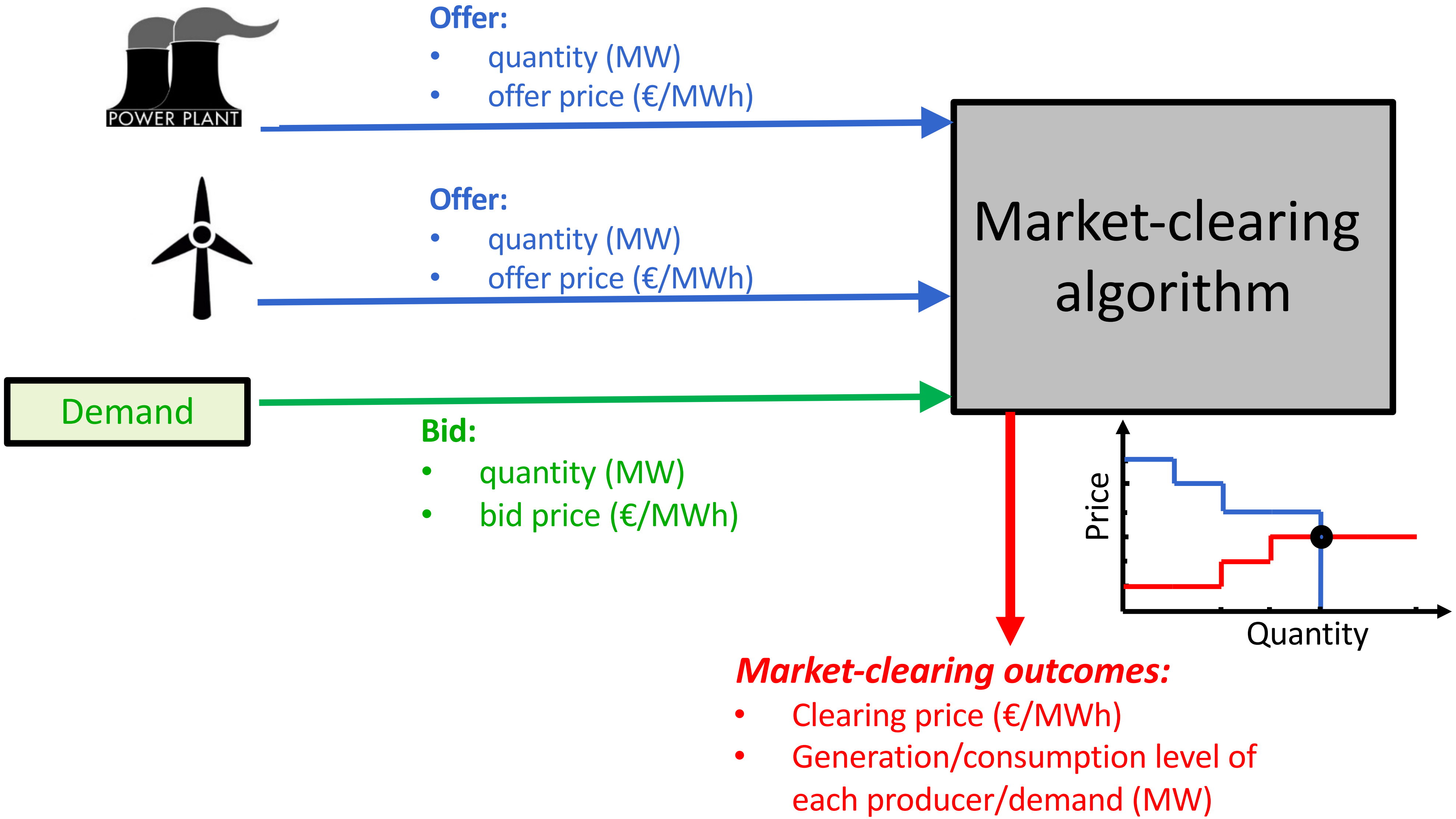
Key topics from the previous lecture:

- **Centralized power systems vs. electricity markets**
- **Supply and demand curves** and their role in market dynamics
- **Merit-order principle**: Least-cost dispatch in the supply side
- **Market clearing**: Determining prices and quantities based on supply and demand curves
- **Social welfare**: Definition and significance in market efficiency
- **Equilibrium concepts**: Market-clearing price and traded quantities
- **Pricing mechanisms**: Uniform pricing vs. pay-as-bid pricing
- **Unique characteristics of electricity markets** compared to other commodity markets

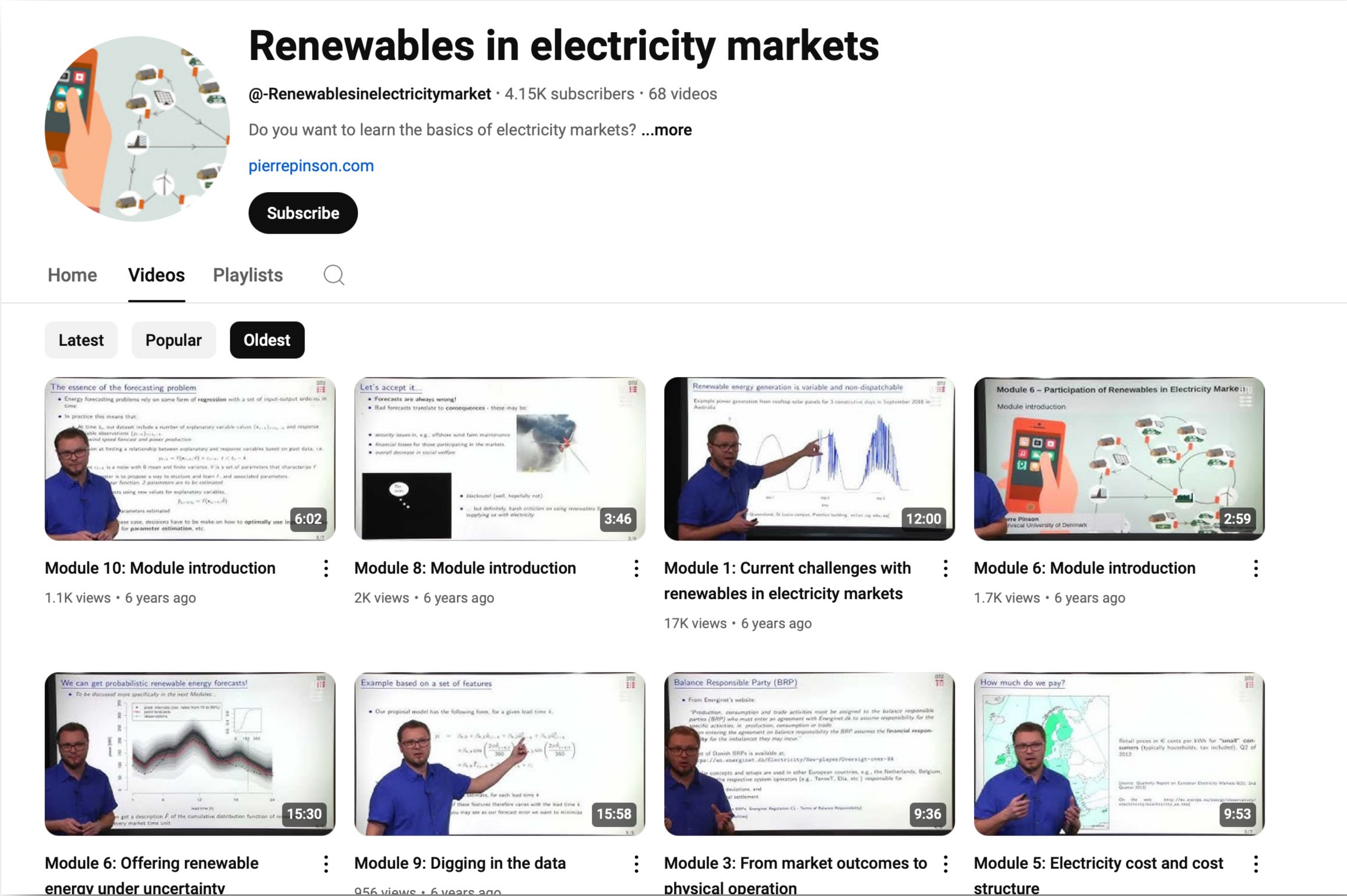
Electricity markets



Electricity markets



An additional resource for learning



By Prof. Pierre Pinson, 2019

https://www.youtube.com/@-Renewablesinelectricitymarket/videos

Learning objectives of this lecture

After this lecture, you will be able to:

- Identify key actors in the electricity market.
- Describe the different types of markets in the power sector.
- Compare and contrast the current practices in European and U.S. electricity markets.
- Formulate the market-clearing procedure as a linear optimization problem, excluding the transmission network.

Outline

- Various market actors
- Various types of electricity market
- European vs. U.S. electricity markets
- Market-clearing as a linear optimization problem

Market actors

- Also referred to as “market players” or “market participants”!

Which market actors have we identified so far?

Market actors: Power producers

Power producers

- A **conventional** generation unit, e.g., coal, gas-fired power plants, combined heat and power (CHP) plants, nuclear plants, etc.
- A **renewable** power unit, e.g., wind, solar, hydro, biomass, etc.
- A producer who owns a **portfolio** of conventional and/or renewable units, with assets potentially distributed across the transmission network.

Examples in Denmark:

Ørsted

VATTENFALL

EUROPEAN ENERGY

HOFOR

CIP
Copenhagen Infrastructure Partners

Market actors: Power demands

Power demands

- Large consumers, e.g., industrial plants
- Retailers

Market actors: Power demands

Power demands

- Large consumers, e.g., industrial plants
- Retailers

A retailer is an intermediate market actor (or trader) who purchases electricity in bulk from electricity markets and sells it to a large number of small-scale consumers, e.g., households.

Market actors: Power demands

Power demands

- Large consumers, e.g., industrial plants
- Retailers

A retailer is an intermediate market actor (or trader) who purchases electricity in bulk from electricity markets and sells it to a large number of small-scale consumers, e.g., households.

Examples in Denmark:

andel energi EWII OK vindstød

Visit Elpris.dk to explore and compare electricity retailers available in your area.

Market actors: Market Operator

Market operator

A non-profit entity that receives all offers from producers and bids from consumers, clears the market by maximizing social welfare, and ultimately disseminates the market-clearing outcomes, including prices and quantities.

Examples in Europe:

Market actors

Do we need any other actor?

Market actors

Do we need any other actor?

Key point:

- The market-clearing outcomes are financial contracts, i.e., contracts for buying and selling electricity.
- Who is responsible for operating the power system in practice based on these contracts?

Market actors: System operator

Transmission System Operator (TSO)

In the context of European electricity markets:

TSOs are responsible for the "safe" operation of the underlying power grid in the **high-voltage transmission level** (typically meshed grid), ensuring supply security, real-time power supply and demand balance, and system stability

Market actors: System operator

Transmission System Operator (TSO)

In the context of European electricity markets:

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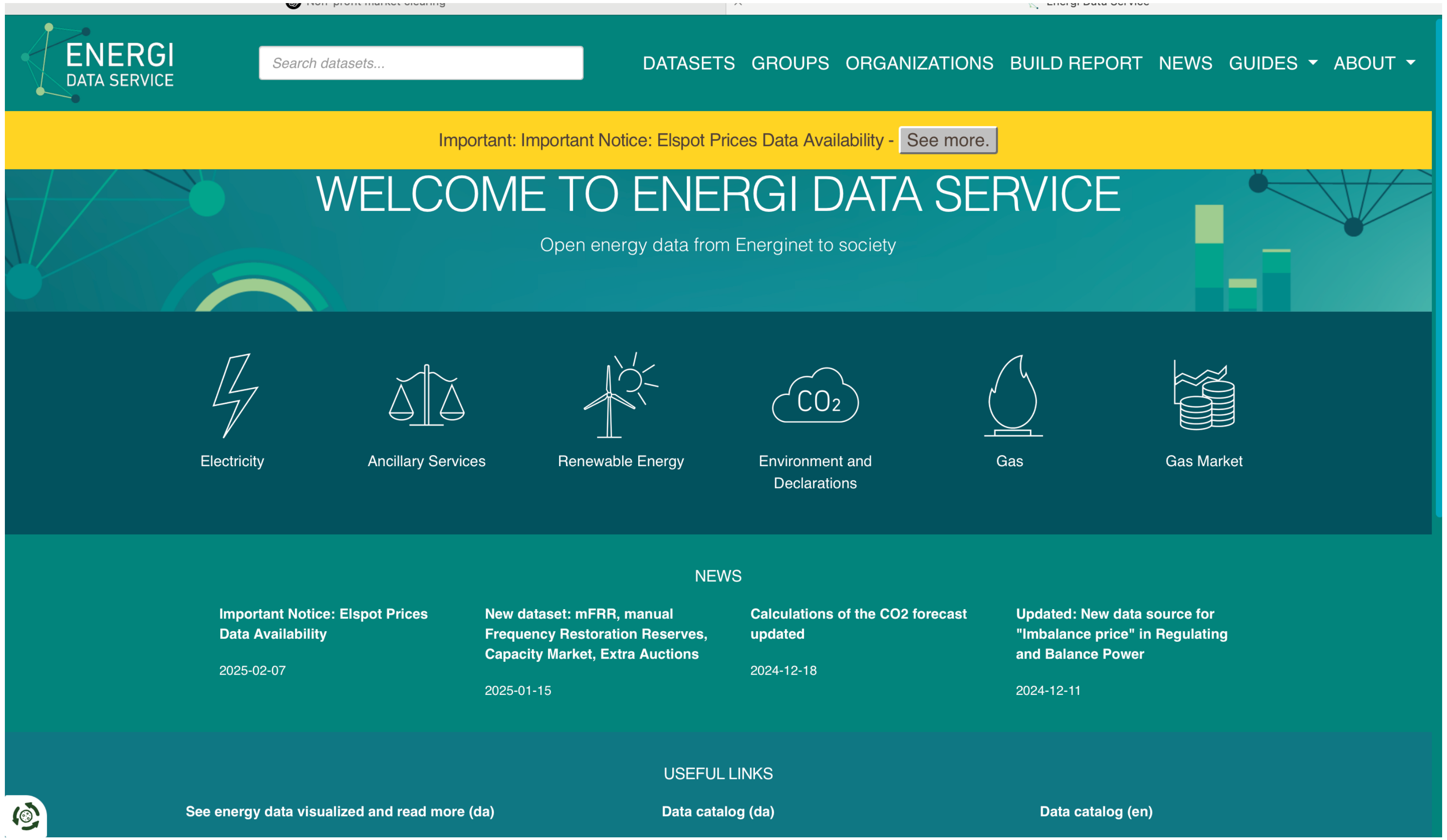
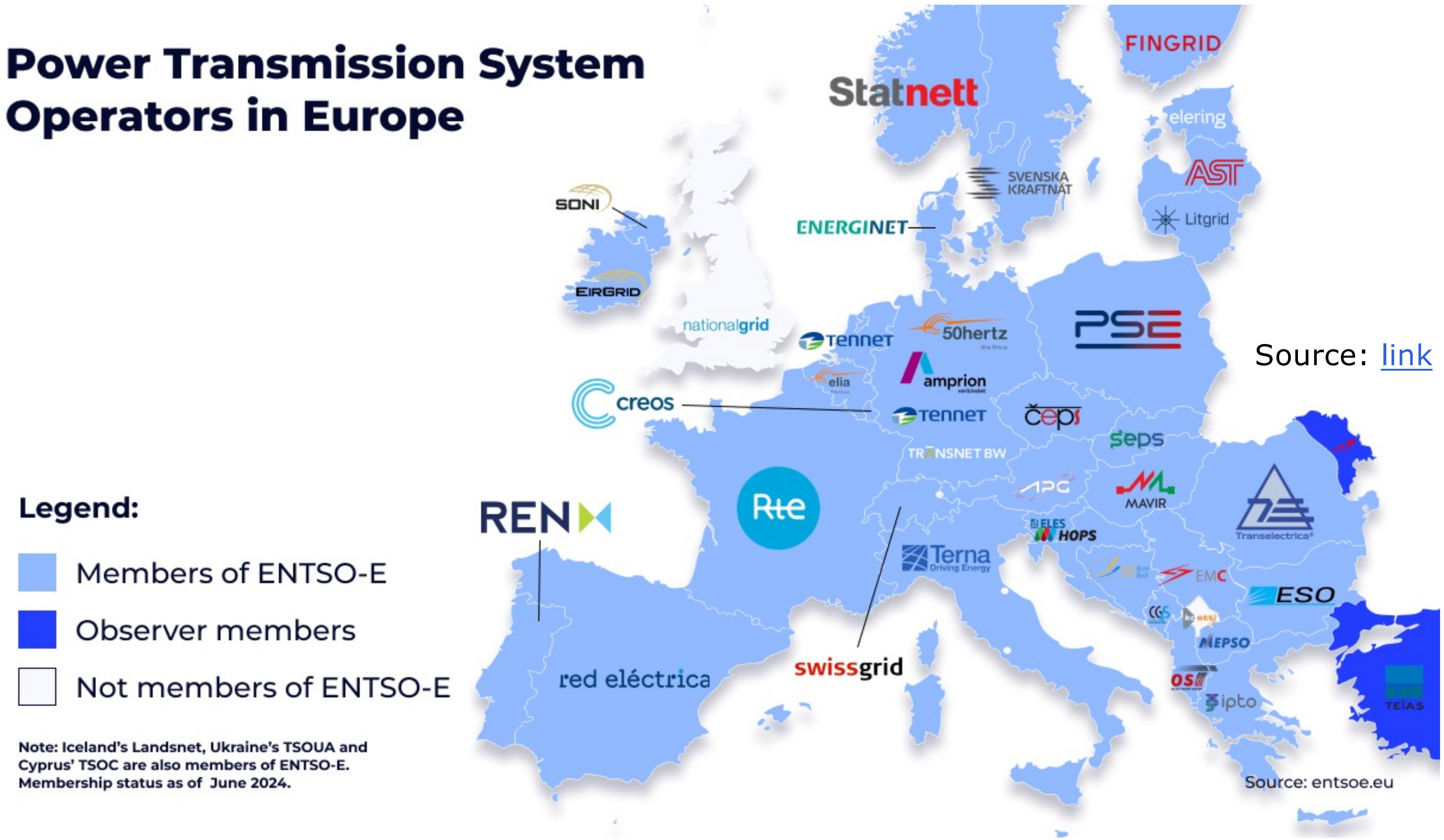
In Denmark, **Energinet** (https://energinet.dk), as the Danish TSO, is responsible for the operation of both the power and gas systems at the transmission level.

Market actors: System operator

Energinet publishes a wealth of important data, including real-time system information:



https://www.energidataservice.dk

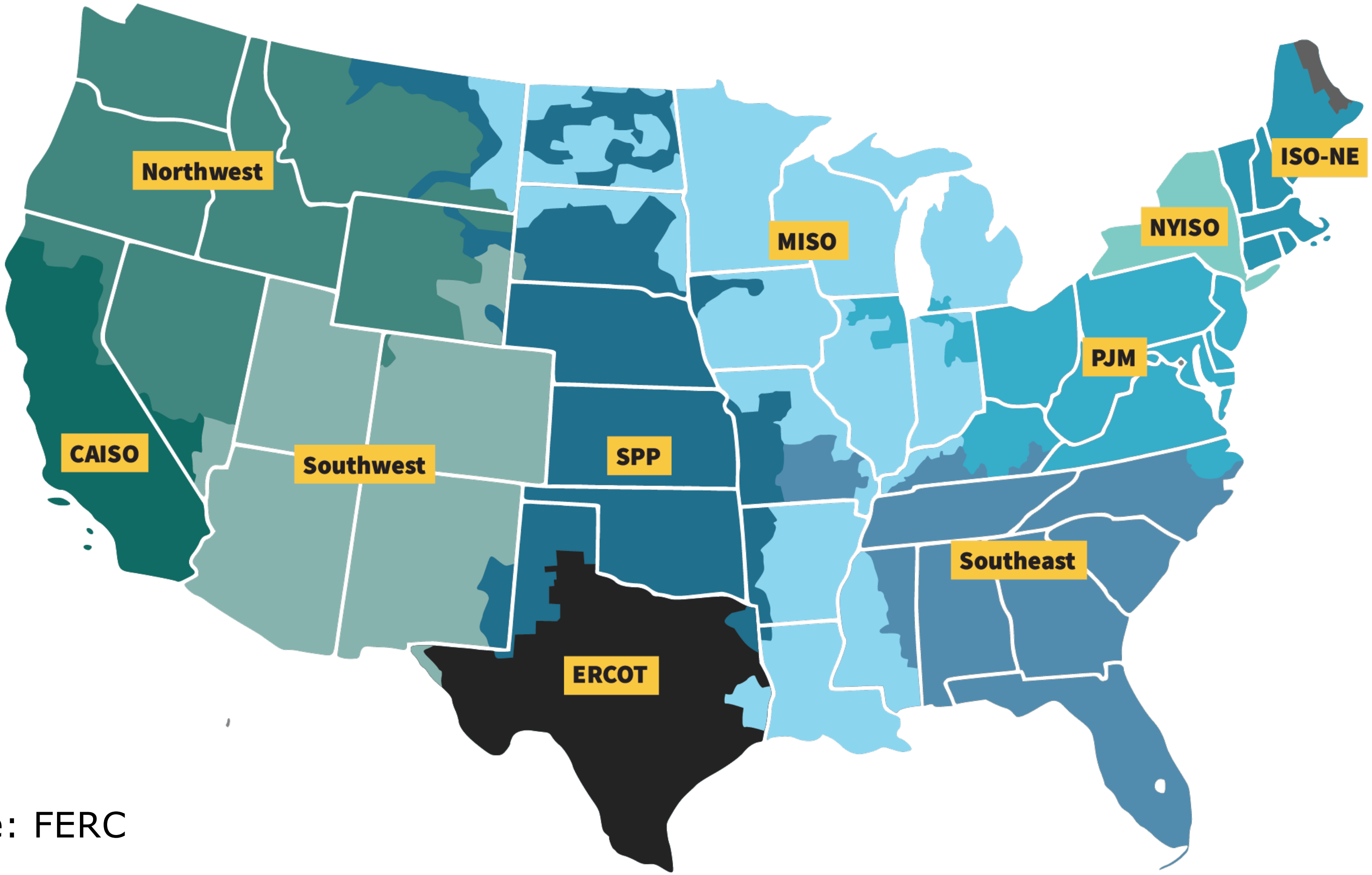


Source: [link]

ENTSO-e: European Network of Transmission System Operators for Electricity (www.entsoe.eu)

Market actors: System operator

In the context of U.S. electricity markets,
this role is typically carried out by the **Independent System Operator (ISO)** or **Regional Transmission Organization (RTO)**, depending on the region.



Source: FERC

Market actors: System operator

Distribution System Operator (DSO)

DSOs are responsible for the "safe" operation of the underlying power grid in the **mid- and low-voltage distribution level** (typically radial grid).

Market actors: System operator

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Examples in Denmark:

Market actors: System operator

Distribution System Operator (DSO)

DSOs are responsible for the "safe" operation of the underlying power grid in the **mid- and low-voltage distribution level** (typically <u>radial</u> grid).

Examples in Denmark:

TREFOR

radius

Elnetselskabet N1

cerius

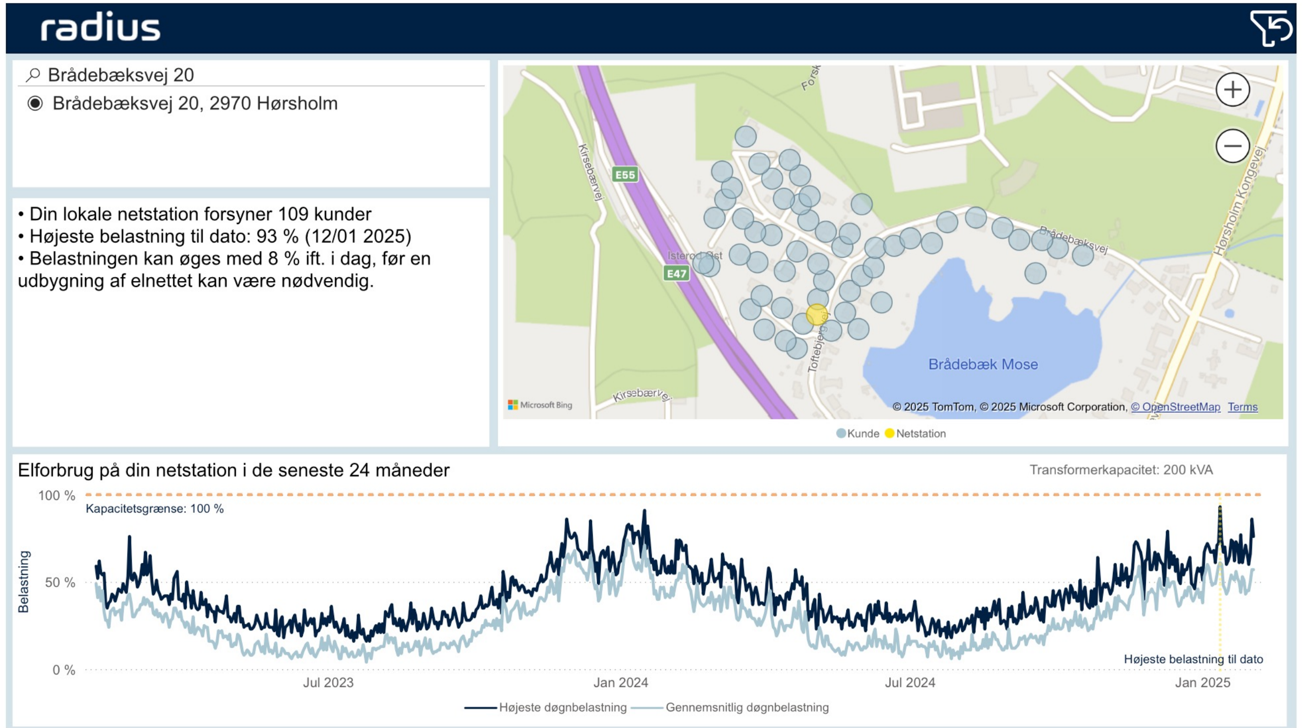
DSOs in Zealand (eastern Denmark)

Market actors: System operator

Distribution System Operator (DSO)

DSOs are responsible for the "safe" operation of the underlying power grid in the **mid- and low-voltage distribution level** (typically radial grid).

An example task of a DSO:

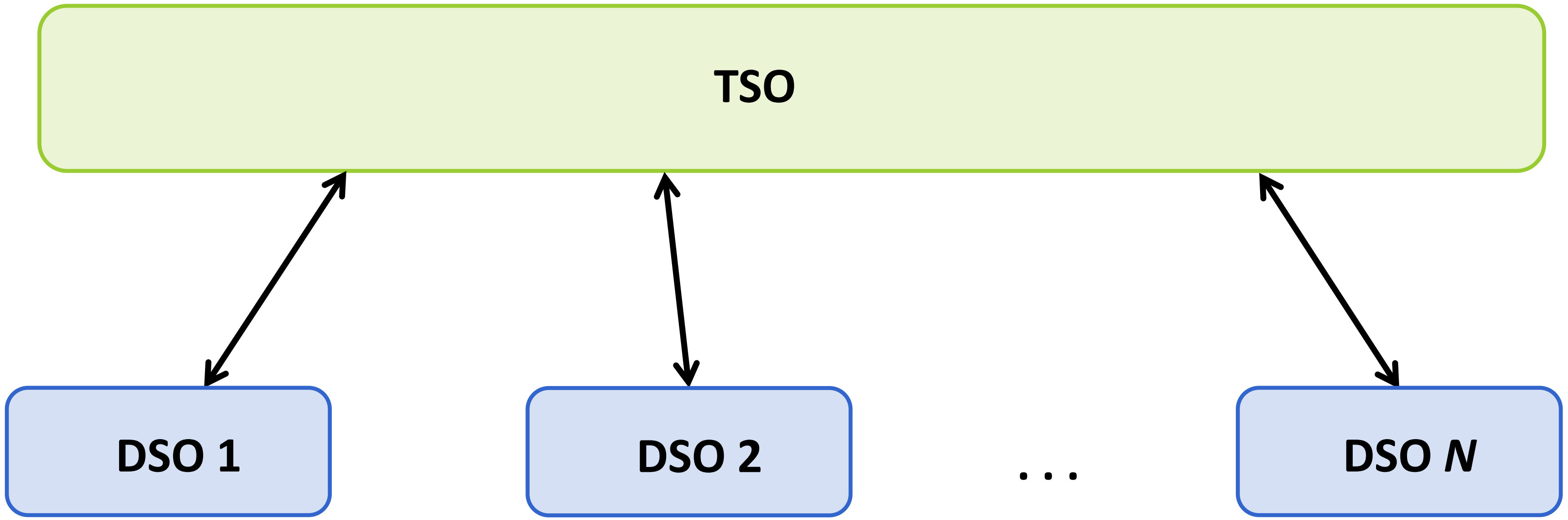


Transformer loading -- Radius: https://radiuselnet.dk/om-elnettet/dit-elnet/

Market actors: System operator

TSO-DSO coordination

Proper coordination between the **TSO** and each **DSO** is essential for ensuring the safe and reliable operation of the entire power grid!



Market actors: Market regulator

Market regulator

Responsible for monitoring market performance in both the short and long run, as well as designing appropriate market regulations and policies.

Examples:

- **The Danish Utility Regulatory** (forsyningstilsynet.dk) in Denmark
- **Federal Energy Regulatory Commission (FERC)** in the U.S.
- ACER
  European Union Agency for the Cooperation of Energy Regulators

Other market actors

- **Traders** (both physical and purely financial),
- **Balancing Responsible Parties (BRPs**),
- **Flexibility Aggregators**,
- and other relevant market actors

will be introduced throughout the course as needed.

Market actors: Recap

- **Power producers**
- **Power demands**
- **Market operator**
- **System operators (TSO, DSO, ISO)**
- **Market regulator**
- **Traders** (both physical and purely financial),
- **Balancing Responsible Parties (BRPs**),
- **Flexibility Aggregators**,
- and other relevant market actors

Outline

- Various market actors
- Various types of electricity market
- European vs. U.S. electricity markets
- Market-clearing as a linear optimization problem

Various markets in the power sector

Three main markets with different products are:

Capacity markets	**Energy markets**	**Ancillary service markets**

Various markets in the power sector

Three main markets with different products are:

Capacity markets | Energy markets | Ancillary service markets

- Capacity markets are designed to ensure that sufficient generation capacity (measured in MW) is available to maintain supply security and support reliable system operation.
- Any producer submitting an offer to the electricity market—regardless of whether it is accepted or rejected—is eligible for compensation based on their availability (measured in €/MW).
- Capacity payments serve as an incentive for power producers to invest in new generation assets over the long term.

However, we will not cover capacity markets in this course.

Various markets in the power sector

Three main markets with different products are:

Capacity markets | **Energy markets** | Ancillary service markets

A central marketplace for exchanging energy (in MWh), where electricity supply and demand are matched.

Various markets in the power sector

Three main markets with different products are:

Capacity markets | **Energy markets** | Ancillary service markets

A central marketplace for exchanging energy (in MWh), where electricity supply and demand are matched.

Various energy markets are cleared at different points in time, including:

Various markets in the power sector

Three main markets with different products are:

Capacity markets | **Energy markets** | Ancillary service markets

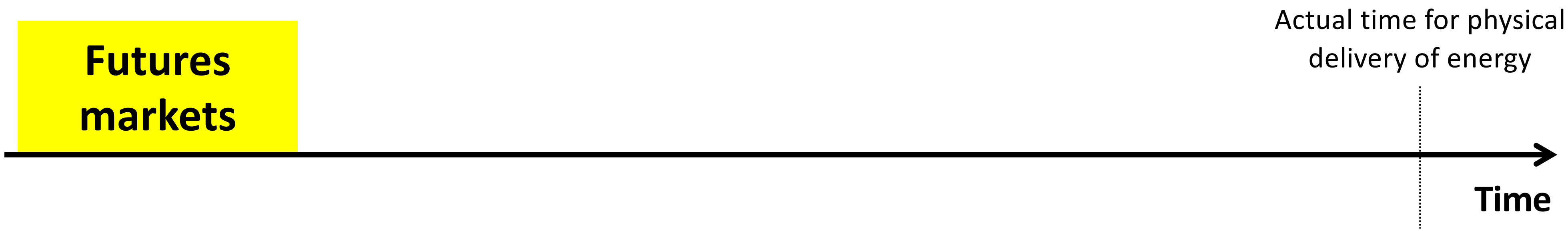
1) Futures markets:

Long-Term Financial Contracts (with a time span of up to six years)

Purpose: Price hedging and risk management for buyers and sellers.

Example: NASDAQ OMX Commodities [link] in Scandinavian countries.

Note: This topic will *not* be covered in this course.



Various markets in the power sector

Three main markets with different products are:

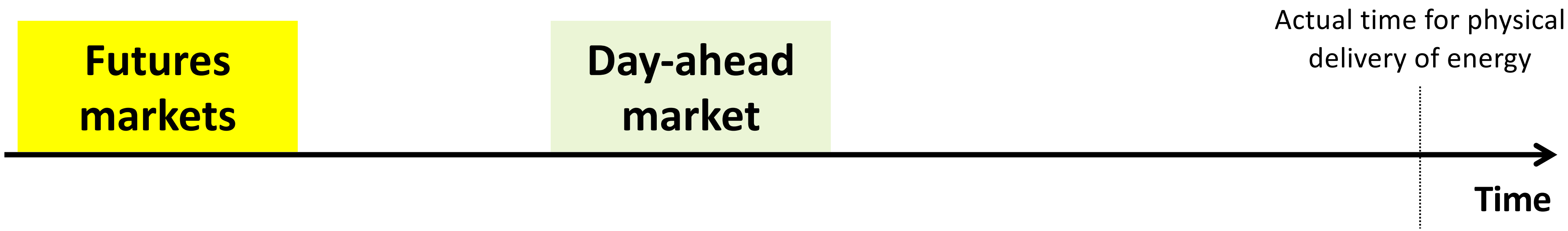
Capacity markets | **Energy markets** | Ancillary service markets

2) Day-ahead market:

also known as the “spot” market

Timing: Cleared 12–36 hours before the actual delivery time.

Example: Nord Pool Elspot, which is cleared at noon on day D-1 for energy exchange during each hour of day D (from midnight to midnight), resulting in different market prices and quantities for each hour.



Various markets in the power sector

Three main markets with different products are:

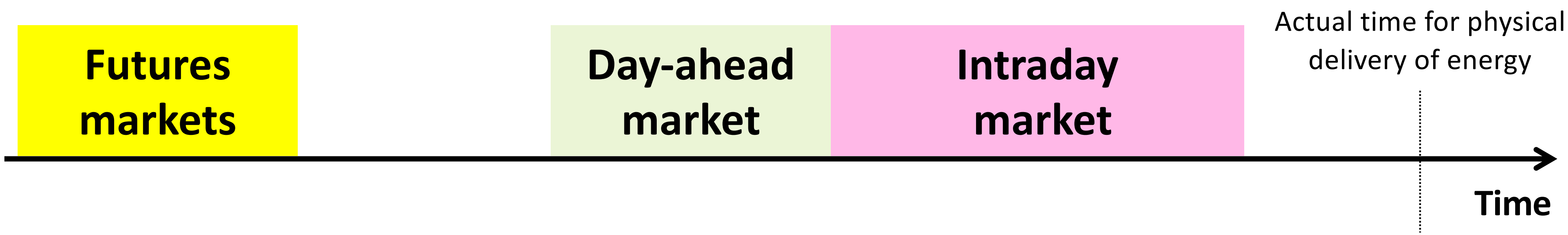
Capacity markets | **Energy markets** | Ancillary service markets

3) Intraday market:

This market provides an opportunity to "modify" day-ahead schedules in case of updated forecasts, asset failures, or for trading purposes. It operates as a continuous and/or discrete auction-based trading platform between the day-ahead market and real-time.

Example: Nord Pool Elbas

It starts at 3 PM on D-1 and ends 1 hour before real-time. Three discrete auction-based sessions (ID-D) and a continuous platform (ID-C) are available. The current time resolution is 1 hour, but it will shift to 15 minutes soon.



Various markets in the power sector

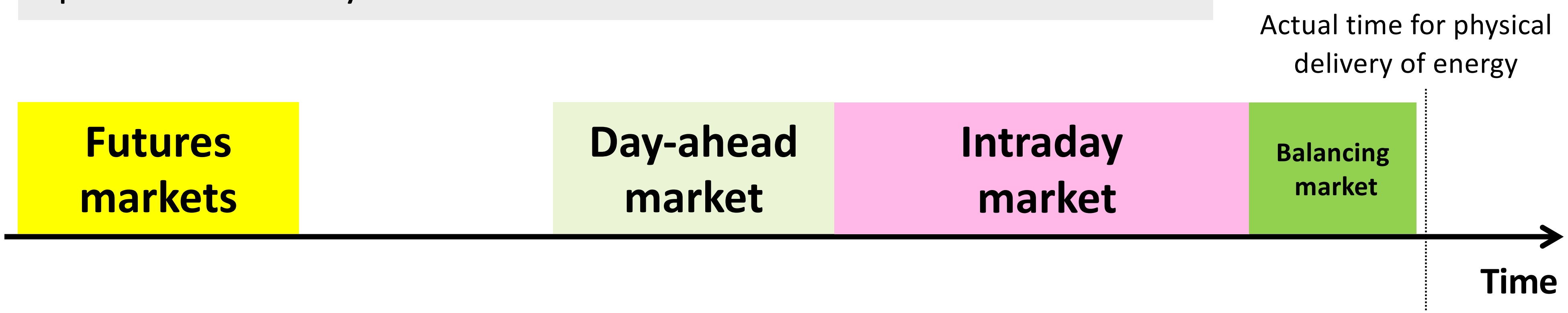
Three main markets with different products are:

Capacity markets | **Energy markets** | Ancillary service markets

4) Balancing market:

- Also known as the "imbalance" market or the "real-time" market.
- This market operates close to real-time system operations.

Purpose: To ensure power supply-demand balance and the safe operation of the system.



Various markets in the power sector

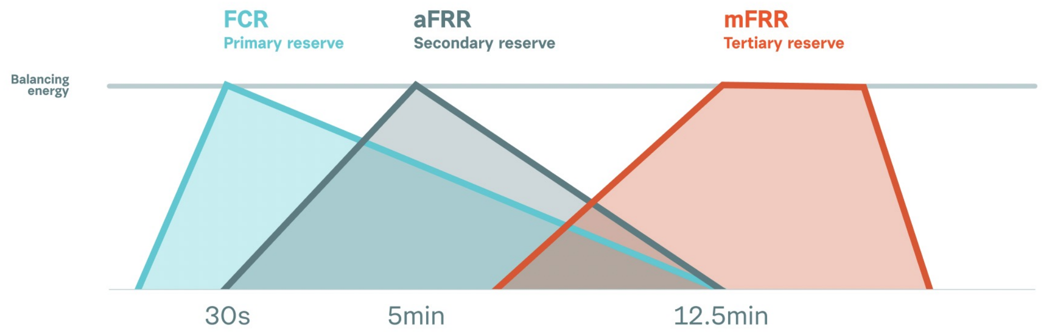
Three main markets with different products are:

Capacity markets | Energy markets | **Ancillary service markets**

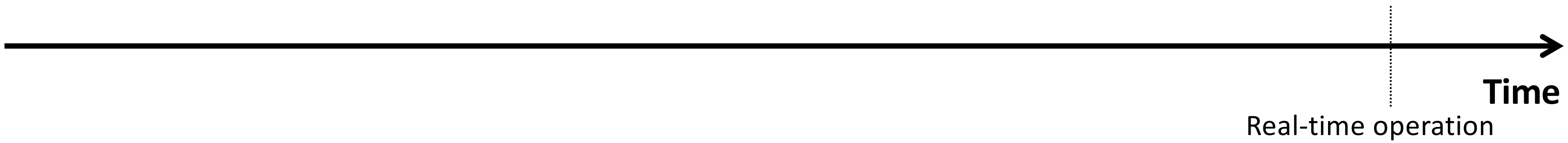
These markets allow the system operator to procure services necessary for the secure and reliable operation of the system.

Examples of such services include:

- Primary reserves
- Secondary reserves
- Tertiary reserves
- Black-start capability
- Reactive and voltage-control reserves
- And more.

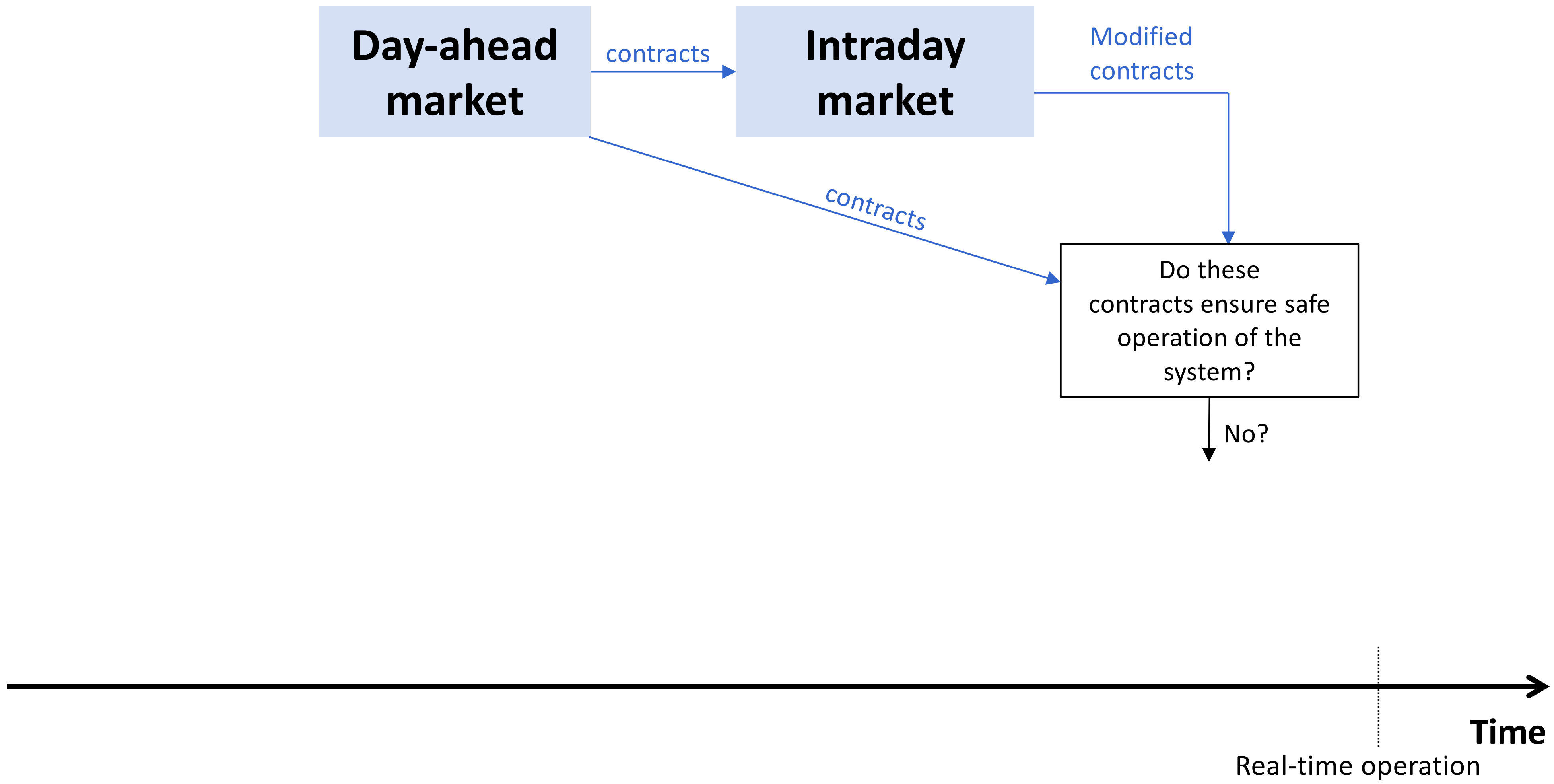


From market to operation



From market to operation

By the market operator (e.g., Nord Pool)



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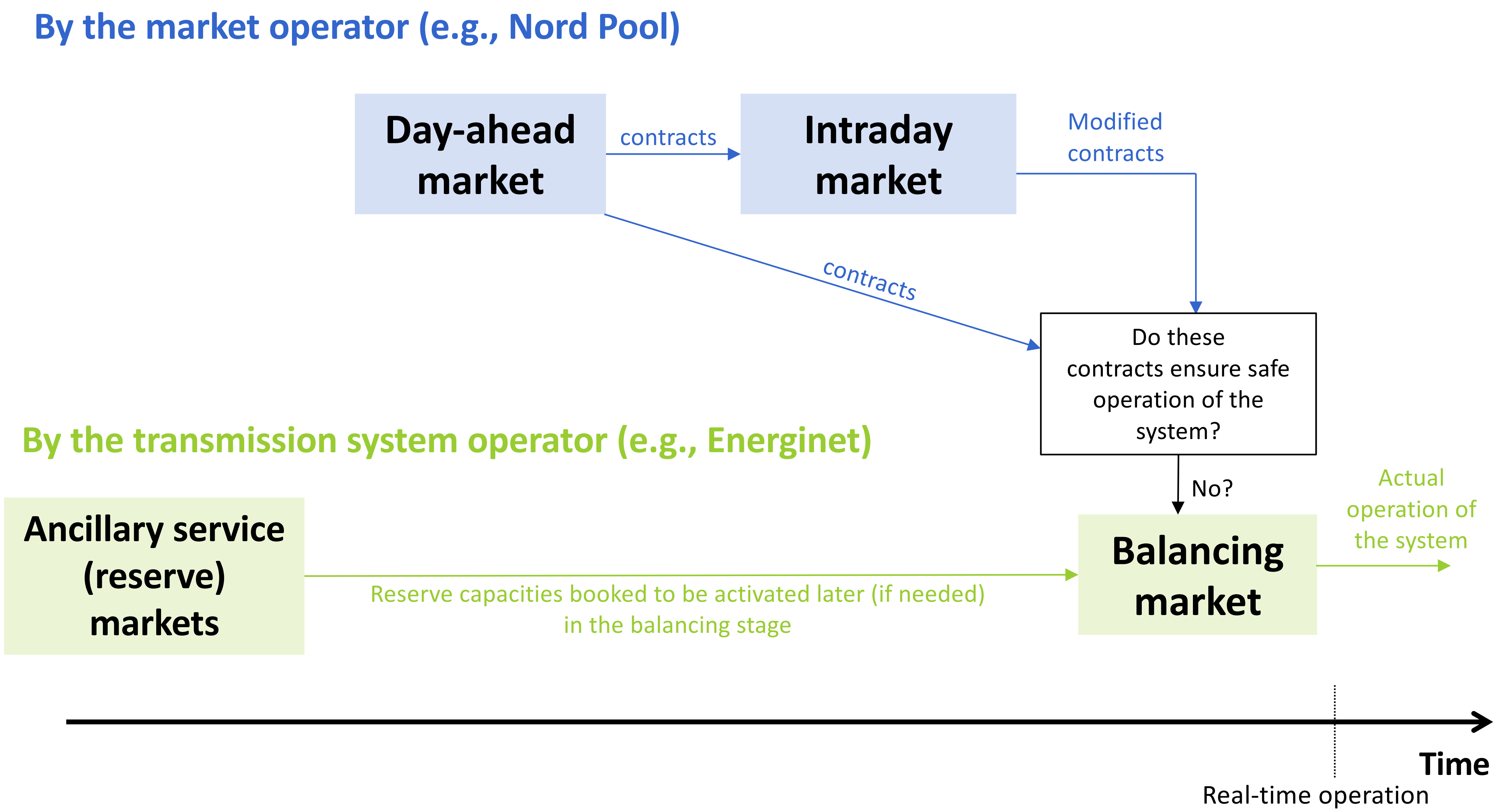
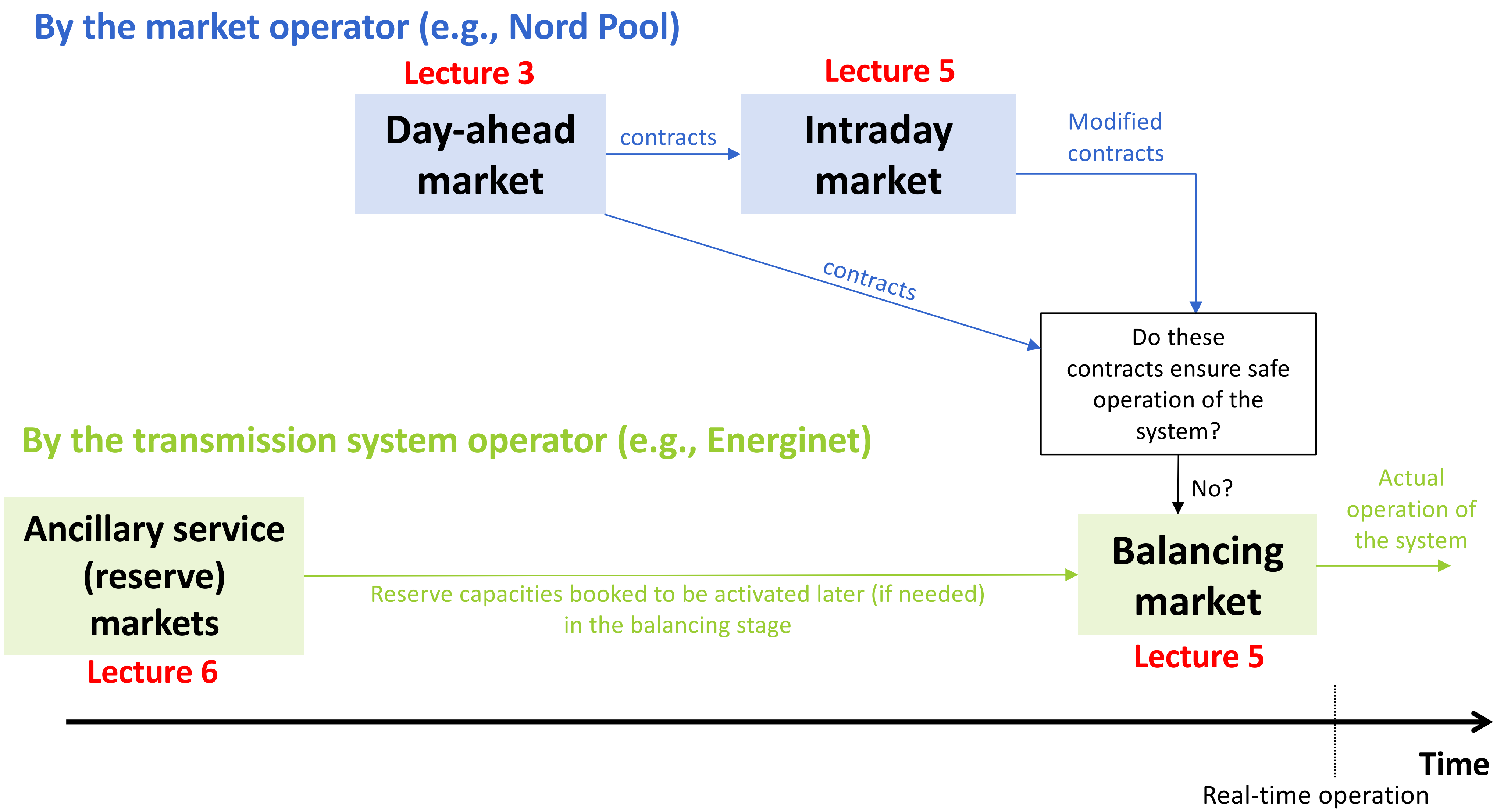


From market to operation



Outline

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European vs U.S. electricity markets

In terms of <u>market and system operation</u>:

In Europe:
Market operators and system operators are **separate** entities (e.g., Nord Pool as the market operator and Energinet as the TSO)

In the U.S.:
An Independent System Operator (ISO) is responsible for **both** clearing the market and operating the system (e.g., CAISO and PJM).

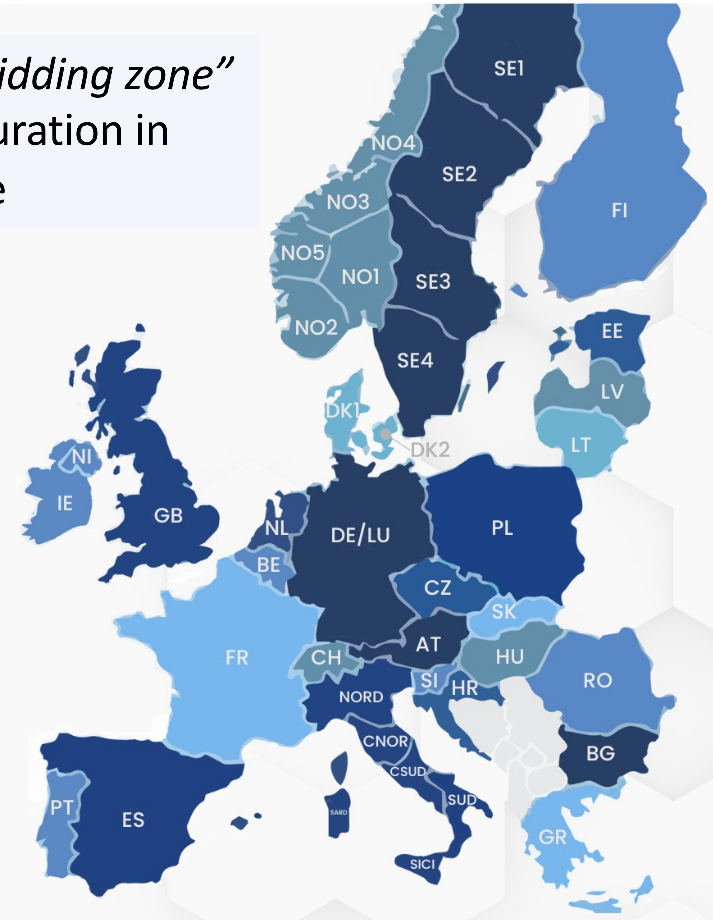
European vs U.S. electricity markets

In terms of <u>network modeling</u> for market clearing:

In Europe:
The transmission network is modeled in a simplified manner (using a **zonal** representation) within the day-ahead and intraday market-clearing process. There is no detailed network modeling within each bidding zone.

In the U.S.:
The transmission network is fully modeled using a **nodal** representation and a linearized power flow model in all market-clearing problems.

The *"bidding zone"* configuration in Europe



Source: [link]

European vs U.S. electricity markets

In terms of energy and reserve markets:

In Europe:
The reserve markets are cleared **separately** by the TSOs (e.g., Energinet) before the day-ahead energy market is cleared by the market operator.

In the U.S.:
A **joint** energy and reserve market exists in the day-ahead time stage, cleared by the ISO, resulting in an energy and reserve dispatch **co-optimization** problem.

European vs U.S. electricity markets

Regarding <u>complex offers</u> versus <u>complex market</u> clearing:

In Europe:

Market actors, such as power producers, are responsible for ensuring their offers/bids adhere to their technical constraints (e.g., ramp limits, minimum production limits, and minimum up/down time limits for conventional generators).

- Market actors **internalize** their technical limits within their own offers/bids:

  → They offer/bid for their entire portfolio within a bidding zone (not per asset).
  → This results in complex orders (bids and offers), while the market-clearing process remains relatively simple.

See "**Block Orders**" in Nord Pool: [link]

Block order

A block order consists of a specified volume and price for a certain number of consecutive hours within the same day. There are four types of block orders defined in Nord Pool markets: regular, profiled, curtailable and linked.

A block order is particularly useful for example if the participant wants to run a power station for a longer period than one hour and wish to minimise costs related to start and stop.

European vs U.S. electricity markets

Regarding <u>complex offers</u> versus <u>complex market</u> clearing:

In U.S.:

Market actors, such as power producers, submit all their technical constraints (e.g., ramp limits, minimum production limits, and minimum up/down time limits for conventional generators) to the market.

- As a result, the market-clearing problem becomes a unit commitment problem with on/off (0/1 binary) variables.

  → This introduces pricing challenges due to the binary variables (not to be discussed in this course).

  → Thus, the market-clearing problem is complex, while the bids and offers remain relatively simple.

Outline

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An illustrative example

Consider an electricity market consisting of two conventional generators (G1 and G2) and one wind farm (W1), where the total demand is price-inelastic.

Production forecast: 20 MW
Offer price: €0/MWh

Capacity: 50 MW
Offer price: €20/MWh

Capacity: 100 MW
Offer price: €30/MWh

An illustrative example

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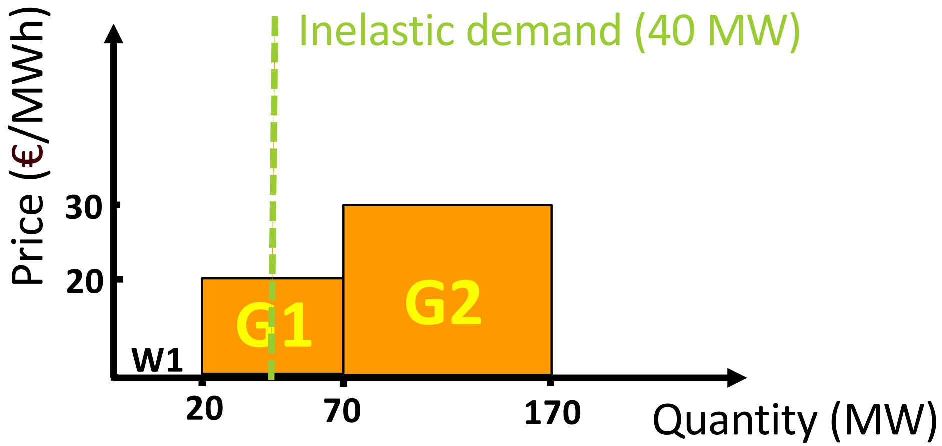

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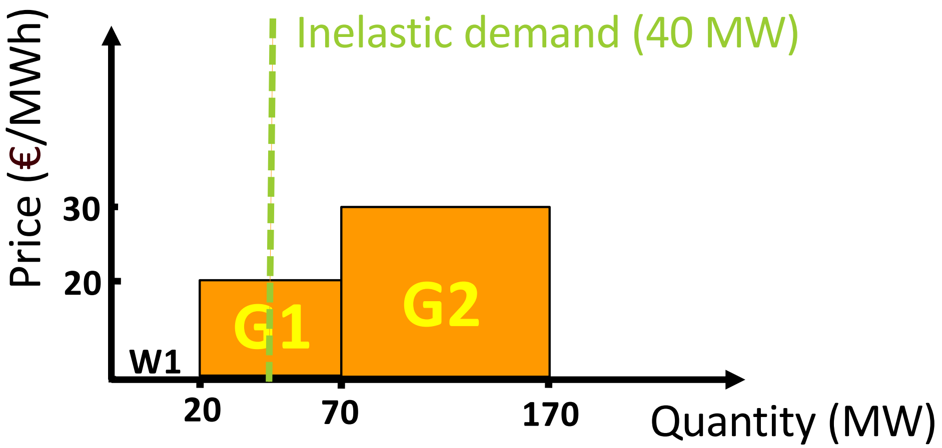
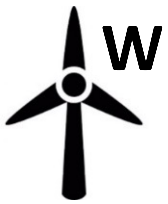


Market-clearing outcomes:

- Production level (MW) of W1: **?**
- Production level (MW) of G1: **?**
- Production level (MW) of G2: **?**
- Market-clearing price (€/MWh): **?**

An illustrative example

Consider an electricity market consisting of two conventional generators (G1 and G2) and one wind farm (W1), where the total demand is price-inelastic.



W1

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Offer price: €0/MWh

Capacity: 50 MW
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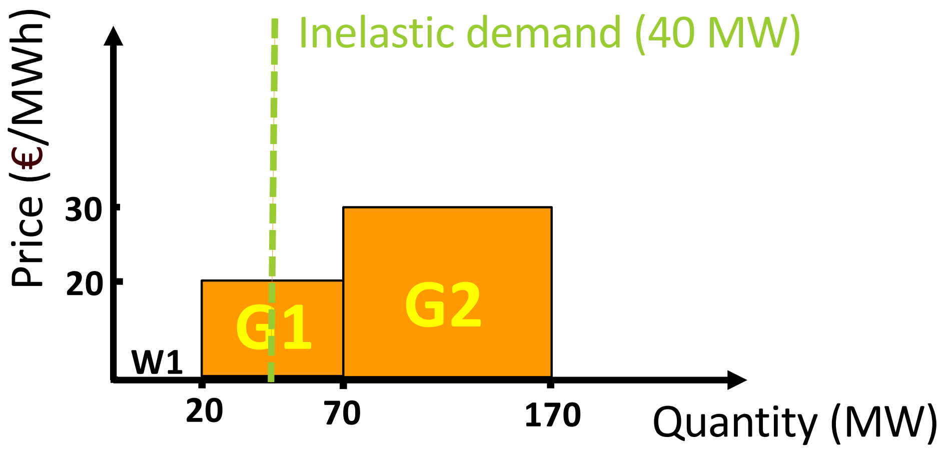
Capacity: 100 MW
Offer price: €30/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **20**
- Production level (MW) of G2: **0**
- Market-clearing price (€/MWh): **20**

Optimization problem for market clearing

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Optimization problem for market clearing

$$\underset{\boxed{p^{W1}, p^{G1}, p^{G2}}}{\text{Minimize}} \quad \boxed{0\ p^{W1} + 20\ p^{G1} + 30\ p^{G2}}$$

Objective function

Set of primal variables

subject to:

$$\boxed{\begin{aligned} & 0 \leq p^{W1} \leq 20 \quad : \quad \underline{\mu}^{W1}, \overline{\mu}^{W1} \\ & 0 \leq p^{G1} \leq 50 \quad : \quad \underline{\mu}^{G1}, \overline{\mu}^{G1} \\ & 0 \leq p^{G2} \leq 100 \quad : \quad \underline{\mu}^{G2}, \overline{\mu}^{G2} \\ & 40 - p^{W1} - p^{G1} - p^{G2} = 0 \quad : \quad \lambda \end{aligned}}$$

Set of constraints
(6 inequality constraints and one equality constraint)

Optimization problem for market clearing

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

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$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \boxed{\lambda}$$

Dual variables,
also referred to as "Lagrange multipliers",
are associated with each constraint.

Optimization problem for market clearing

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

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$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \boxed{\lambda}$$

Questions:

- What does a dual variable represent (mathematical interpretation)?
- Can we infer anything about the sign of different dual variables?

Dual variables,
also referred to as "Lagrange multipliers",
are associated with each constraint.

Optimization problem for market clearing

$$\underset{p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Minimize}} \quad \underbrace{0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}}_{\substack{\text{Objective function:} \\ \text{Total power generation cost of the system}}}$$

subject to:

$$\left.\begin{aligned}
&0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}} \\
&0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}} \\
&0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}
\end{aligned}\right\} \text{Generation limits}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda \longrightarrow \text{Power balance}$$

Optimization problem for market clearing

$$\underset{p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

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$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \boxed{\lambda}$$

The dual variable associated with the power balance equality represents the market-clearing price under a uniform pricing scheme.

Optimization problem for market clearing

$$\underset{p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}}{\text{Minimize}} \quad 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \boxed{\lambda}$$

Question:

How can we compute the optimal value of this dual variable?

The dual variable associated with the power balance equality represents the market-clearing price under a uniform pricing scheme.

How to derive Lagrangian function?

How to derive Lagrangian function?

$$
\begin{aligned}
&\underset{x}{\text{Minimize}} \quad f(x) \\
&\text{subject to:} \\
&h(x) = 0 \quad : \quad \lambda \\
&g(x) \leq 0 \quad : \quad \mu
\end{aligned}
$$

This is a standard form of an optimization problem!

How to derive Lagrangian function?

$$\begin{aligned} &\underset{x}{\text{Minimize}} \quad f(x) \\ &\text{subject to:} \\ &h(x) = 0 \quad : \quad \lambda \\ &g(x) \leq 0 \quad : \quad \mu \end{aligned}$$

This is a standard form of an optimization problem!

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^\top h(x) + \mu^\top g(x)$$

How to derive Lagrangian function?

Original (primal) problem

$$\underset{x}{\text{Minimize}} \quad f(x)$$

subject to:

$$h(x) = 0 \quad : \quad \lambda$$

$$g(x) \leq 0 \quad : \quad \mu$$

Lagrangian function

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^\top h(x) + \mu^\top g(x)$$

How to derive Lagrangian function?

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$$\underset{x}{\text{Minimize}} \quad f(x)$$

subject to:

$$h(x) = 0 \quad : \quad \lambda$$

$$g(x) \leq 0 \quad : \quad \mu$$

Lagrangian function

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^\top h(x) + \mu^\top g(x)$$

$$\boxed{\begin{aligned} &\frac{\partial \mathcal{L}(x, \lambda, \mu)}{\partial x} = 0 \\ &h(x) = 0 \\ &0 \leq -g(x) \perp \mu \geq 0 \\ &\lambda \in \text{free} \end{aligned}}$$

Optimality
Karush–Kuhn–Tucker (KKT)
conditions

How to derive Lagrangian function?

Original (primal) problem

$$\underset{x}{\text{Minimize}} \quad f(x)$$
subject to:
$$h(x) = 0 \quad : \quad \lambda$$
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Lagrangian function

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$$\frac{\partial \mathcal{L}(x, \lambda, \mu)}{\partial x} = 0$$
$$h(x) = 0$$
$$0 \leq -g(x) \perp \mu \geq 0$$
$$\lambda \in \text{free}$$

Optimality
Karush–Kuhn–Tucker (KKT)
conditions

Complementary slackness,
enforcing g(x) times \mu = 0

Recall the optimization problem for market clearing

Optimization problem for market clearing

Our optimization problem

$$\underset{p^{W1},p^{G1},p^{G2}}{\text{Minimize}} \quad 0\ p^{W1} + 20\ p^{G1} + 30\ p^{G2}$$

subject to:

$$0 \le p^{W1} \le 20 \quad : \quad \underline{\mu}^{W1}, \overline{\mu}^{W1}$$

$$0 \le p^{G1} \le 50 \quad : \quad \underline{\mu}^{G1}, \overline{\mu}^{G1}$$

$$0 \le p^{G2} \le 100 \quad : \quad \underline{\mu}^{G2}, \overline{\mu}^{G2}$$

$$40 - p^{W1} - p^{G1} - p^{G2} = 0 \quad : \quad \lambda$$

Rewritten to a standard form

$$\underset{p^{W1},p^{G1},p^{G2}}{\text{Minimize}} \quad 0\ p^{W1} + 20\ p^{G1} + 30\ p^{G2}$$

subject to:

$$-p^{W1} \le 0 \quad : \quad \underline{\mu}^{W1}$$

$$p^{W1} - 20 \le 0 \quad : \quad \overline{\mu}^{W1}$$

$$-p^{G1} \le 0 \quad : \quad \underline{\mu}^{G1}$$

$$p^{G1} - 50 \le 0 \quad : \quad \overline{\mu}^{G1}$$

$$-p^{G2} \le 0 \quad : \quad \underline{\mu}^{G2}$$

$$p^{G2} - 100 \le 0 \quad : \quad \overline{\mu}^{G2}$$

$$40 - p^{W1} - p^{G1} - p^{G2} = 0 \quad : \quad \lambda$$

Lagrangian function of the market-clearing optimization problem

Standard form of our optimization problem

$$\underset{p^{W1},p^{G1},p^{G2}}{\text{Minimize}} \quad 0\ p^{W1} + 20\ p^{G1} + 30\ p^{G2}$$

subject to:

$$-p^{W1} \leq 0 \quad : \quad \underline{\mu}^{W1}$$
$$p^{W1} - 20 \leq 0 \quad : \quad \overline{\mu}^{W1}$$
$$-p^{G1} \leq 0 \quad : \quad \underline{\mu}^{G1}$$
$$p^{G1} - 50 \leq 0 \quad : \quad \overline{\mu}^{G1}$$
$$-p^{G2} \leq 0 \quad : \quad \underline{\mu}^{G2}$$
$$p^{G2} - 100 \leq 0 \quad : \quad \overline{\mu}^{G2}$$
$$40 - p^{W1} - p^{G1} - p^{G2} = 0 \quad : \quad \lambda$$

Lagrangian function

$$\mathcal{L}(p^{W1}, p^{G1}, p^{G2}, \underline{\mu}^{W1}, \overline{\mu}^{W1}, \underline{\mu}^{G1}, \overline{\mu}^{G1}, \underline{\mu}^{G2}, \overline{\mu}^{G2}, \lambda) =$$
$$\begin{aligned} & 0\ p^{W1} + 20\ p^{G1} + 30\ p^{G2} \\ & - p^{W1}\underline{\mu}^{W1} + (p^{W1} - 20)\,\overline{\mu}^{W1} \\ & - p^{G1}\underline{\mu}^{G1} + (p^{G1} - 50)\,\overline{\mu}^{G1} \\ & - p^{G2}\underline{\mu}^{G2} + (p^{G2} - 100)\,\overline{\mu}^{G2} \\ & + \lambda\left(40 - p^{W1} - p^{G1} - p^{G2}\right) \end{aligned}$$

KKT conditions of the market-clearing optimization problem

$$
\begin{aligned}
\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) = \\
0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\
- p^{\mathrm{W1}} \underline{\mu}^{\mathrm{W1}} + (p^{\mathrm{W1}} - 20)\, \overline{\mu}^{\mathrm{W1}} \\
- p^{\mathrm{G1}} \underline{\mu}^{\mathrm{G1}} + (p^{\mathrm{G1}} - 50)\, \overline{\mu}^{\mathrm{G1}} \\
- p^{\mathrm{G2}} \underline{\mu}^{\mathrm{G2}} + (p^{\mathrm{G2}} - 100)\, \overline{\mu}^{\mathrm{G2}} \\
+ \lambda \left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)
\end{aligned}
$$

$\Downarrow$ KKT conditions

KKT conditions of the market-clearing optimization problem

$$\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) =$$

$$\begin{aligned}
&0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\
&- p^{\mathrm{W1}} \underline{\mu}^{\mathrm{W1}} + (p^{\mathrm{W1}} - 20)\, \overline{\mu}^{\mathrm{W1}} \\
&- p^{\mathrm{G1}} \underline{\mu}^{\mathrm{G1}} + (p^{\mathrm{G1}} - 50)\, \overline{\mu}^{\mathrm{G1}} \\
&- p^{\mathrm{G2}} \underline{\mu}^{\mathrm{G2}} + (p^{\mathrm{G2}} - 100)\, \overline{\mu}^{\mathrm{G2}} \\
&+ \lambda \left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)
\end{aligned}$$

KKT conditions

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{W1}}} = 0 + \overline{\mu}^{\mathrm{W1}} - \underline{\mu}^{\mathrm{W1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G1}}} = 20 + \overline{\mu}^{\mathrm{G1}} - \underline{\mu}^{\mathrm{G1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G2}}} = 30 + \overline{\mu}^{\mathrm{G2}} - \underline{\mu}^{\mathrm{G2}} - \lambda = 0$$

$$40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0$$

$$0 \le \left(20 - p^{\mathrm{W1}}\right) \perp \overline{\mu}^{\mathrm{W1}} \ge 0$$

$$0 \le p^{\mathrm{W1}} \perp \underline{\mu}^{\mathrm{W1}} \ge 0$$

$$0 \le \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \ge 0$$

$$0 \le p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \ge 0$$

$$0 \le \left(100 - p^{\mathrm{G2}}\right) \perp \overline{\mu}^{\mathrm{G2}} \ge 0$$

$$0 \le p^{\mathrm{G2}} \perp \underline{\mu}^{\mathrm{G2}} \ge 0$$

KKT conditions of the market-clearing optimization problem

$$
\begin{aligned}
\mathcal{L}(p^{\mathrm{W1}}, p^{\mathrm{G1}}, p^{\mathrm{G2}}, \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}, \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}, \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}, \lambda) = \\
& 0\ p^{\mathrm{W1}} + 20\ p^{\mathrm{G1}} + 30\ p^{\mathrm{G2}} \\
& - p^{\mathrm{W1}} \underline{\mu}^{\mathrm{W1}} + (p^{\mathrm{W1}} - 20)\, \overline{\mu}^{\mathrm{W1}} \\
& - p^{\mathrm{G1}} \underline{\mu}^{\mathrm{G1}} + (p^{\mathrm{G1}} - 50)\, \overline{\mu}^{\mathrm{G1}} \\
& - p^{\mathrm{G2}} \underline{\mu}^{\mathrm{G2}} + (p^{\mathrm{G2}} - 100)\, \overline{\mu}^{\mathrm{G2}} \\
& + \lambda \left(40 - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}}\right)
\end{aligned}
$$

KKT conditions

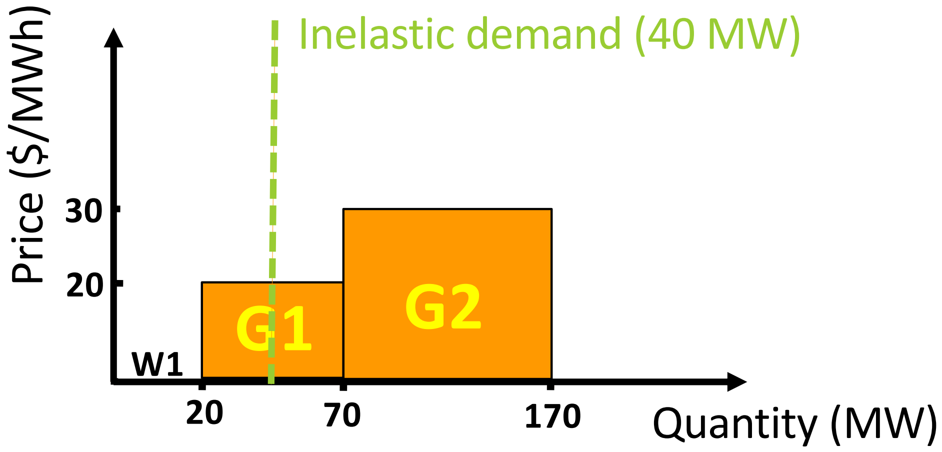
We can solve KKT conditions as a system of equations!

$$
\begin{aligned}
\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{W1}}} &= 0 + \overline{\mu}^{\mathrm{W1}} - \underline{\mu}^{\mathrm{W1}} - \lambda = 0 \\
\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G1}}} &= 20 + \overline{\mu}^{\mathrm{G1}} - \underline{\mu}^{\mathrm{G1}} - \lambda = 0 \\
\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G2}}} &= 30 + \overline{\mu}^{\mathrm{G2}} - \underline{\mu}^{\mathrm{G2}} - \lambda = 0 \\
40 - p^{\mathrm{W1}} &- p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0
\end{aligned}
$$

$$
\begin{aligned}
& 0 \le \left(20 - p^{\mathrm{W1}}\right) \perp \overline{\mu}^{\mathrm{W1}} \ge 0 \\
& 0 \le p^{\mathrm{W1}} \perp \underline{\mu}^{\mathrm{W1}} \ge 0 \\
& 0 \le \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \ge 0 \\
& 0 \le p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \ge 0 \\
& 0 \le \left(100 - p^{\mathrm{G2}}\right) \perp \overline{\mu}^{\mathrm{G2}} \ge 0 \\
& 0 \le p^{\mathrm{G2}} \perp \underline{\mu}^{\mathrm{G2}} \ge 0
\end{aligned}
$$

How to verify the market-clearing price by the KKT conditions?

By solving the market-clearing optimization problem, we determine that the optimal schedule for G1 is 20 MW.



Note: We choose G1 for price verification because it is the **marginal producer,** i.e., the most expensive generator that is either partially or fully dispatched.

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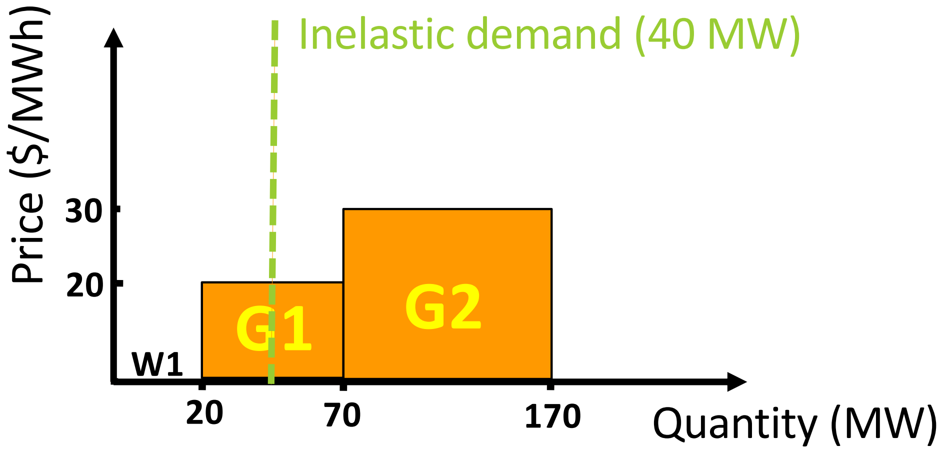

Recall the complementary slackness conditions associated with G1:

$$0 \leq \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \geq 0$$
$$0 \leq p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \geq 0$$

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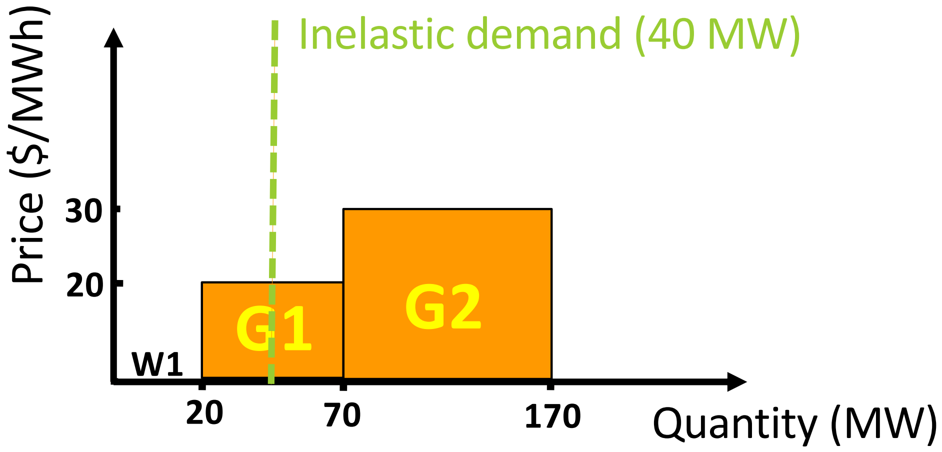
By solving the market-clearing optimization problem, we determine that the optimal schedule for G1 is 20 MW.



Recall the complementary slackness conditions associated with G1:

$$0 \leq \left(50 - p^{\mathrm{G1}}\right) \perp \overline{\mu}^{\mathrm{G1}} \geq 0$$
$$0 \leq p^{\mathrm{G1}} \perp \underline{\mu}^{\mathrm{G1}} \geq 0$$

$$\Rightarrow \quad \underline{\mu}^{\mathrm{G1}} = 0, \quad \overline{\mu}^{\mathrm{G1}} = 0$$

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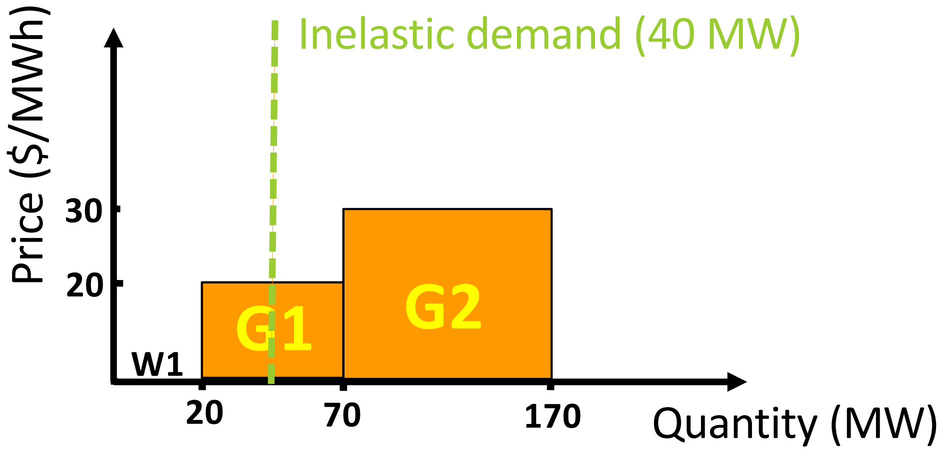
By solving the market-clearing optimization problem, we determine that the optimal schedule for G1 is 20 MW.



Recall the complementary slackness conditions associated with G1:

$$0 \leq \left(50 - p^{\text{G1}}\right) \perp \overline{\mu}^{\text{G1}} \geq 0$$
$$0 \leq p^{\text{G1}} \perp \underline{\mu}^{\text{G1}} \geq 0$$

$\Rightarrow$

$$\underline{\mu}^{\text{G1}} = 0$$
$$\overline{\mu}^{\text{G1}} = 0$$

Recall this KKT condition related to G1:

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\text{G1}}} = 20 + \overline{\mu}^{\text{G1}} - \underline{\mu}^{\text{G1}} - \lambda = 0$$

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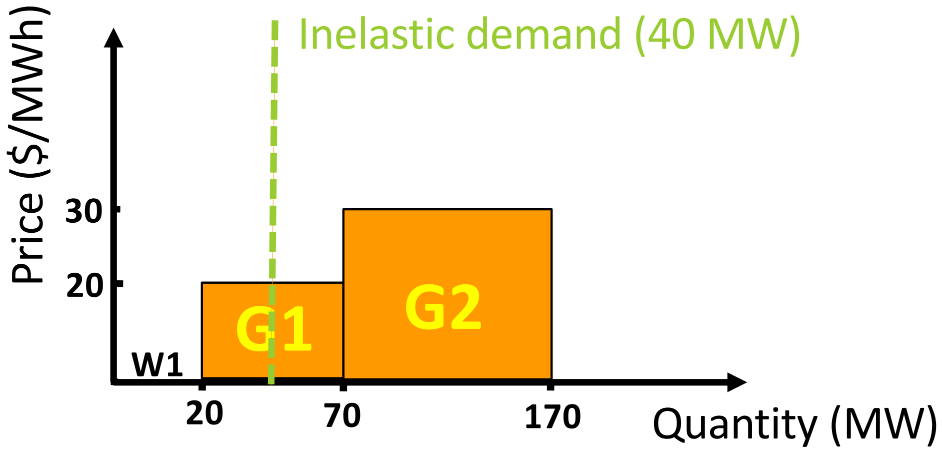
By solving the market-clearing optimization problem, we determine that the optimal schedule for G1 is 20 MW.



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$$\Rightarrow \quad \underline{\mu}^{\mathrm{G1}} = 0, \quad \overline{\mu}^{\mathrm{G1}} = 0$$

Recall this KKT condition related to G1:

$$\frac{\partial \mathcal{L}(.)}{\partial p^{\mathrm{G1}}} = 20 + \overline{\mu}^{\mathrm{G1}} - \underline{\mu}^{\mathrm{G1}} - \lambda = 0 \quad \Rightarrow \quad \lambda = 20$$

This confirms that the market-clearing price is €20/MWh!

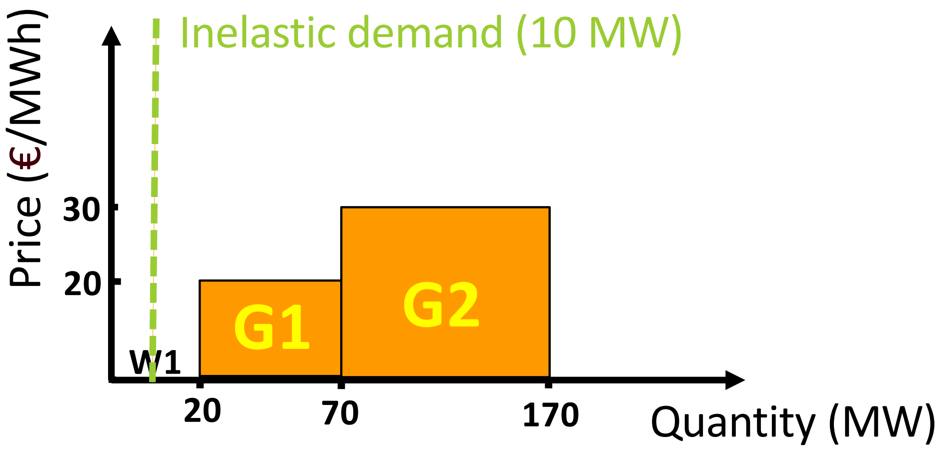
An illustrative example: Different demand

Consider an electricity market consisting of two conventional generators (G1 and G2) and one wind farm (W1), where the total demand is price-inelastic.

Production forecast: 20 MW
Offer price: €0/MWh

Capacity: 50 MW
Offer price: €20/MWh

Capacity: 100 MW
Offer price: €30/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **10**
- Production level (MW) of G1: **0**
- Production level (MW) of G2: **0**
- Market-clearing price (€/MWh): **0**

Verify by the KKT conditions!

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Consider an electricity market consisting of two conventional generators (G1 and G2) and one wind farm (W1), where the total demand is price-inelastic.

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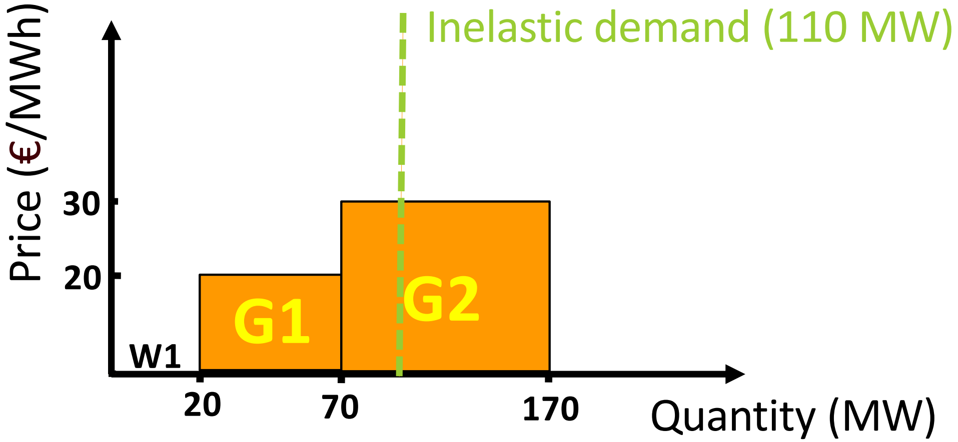
Capacity: 100 MW
Offer price: €30/MWh



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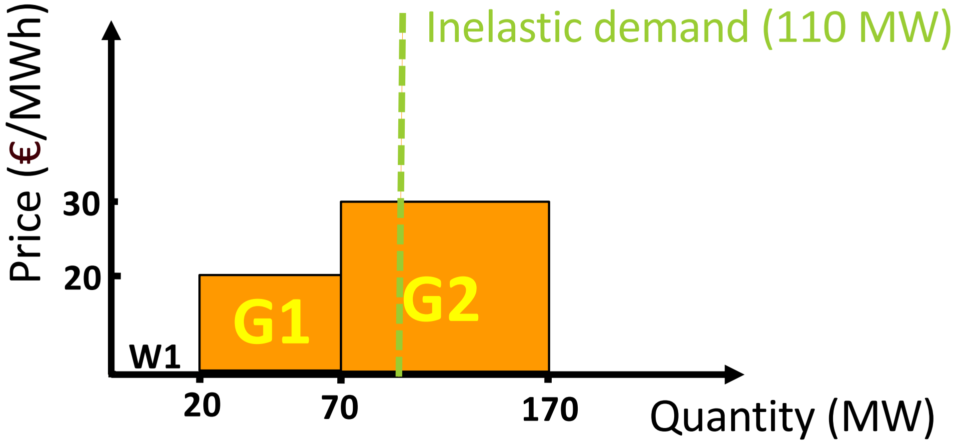
Offer price: €20/MWh

Capacity: 100 MW
Offer price: €30/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **50**
- Production level (MW) of G2: **40**
- Market-clearing price (€/MWh): **30**

Verify by the KKT conditions!

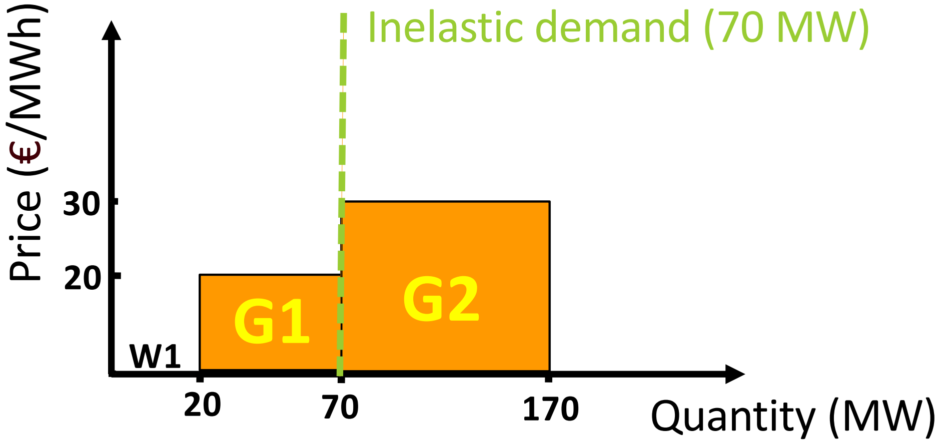
An illustrative example: Different demand

Consider an electricity market consisting of two conventional generators (G1 and G2) and one wind farm (W1), where the total demand is price-inelastic.

Production forecast: 20 MW
Offer price: €0/MWh

Capacity: 50 MW
Offer price: €20/MWh

Capacity: 100 MW
Offer price: €30/MWh



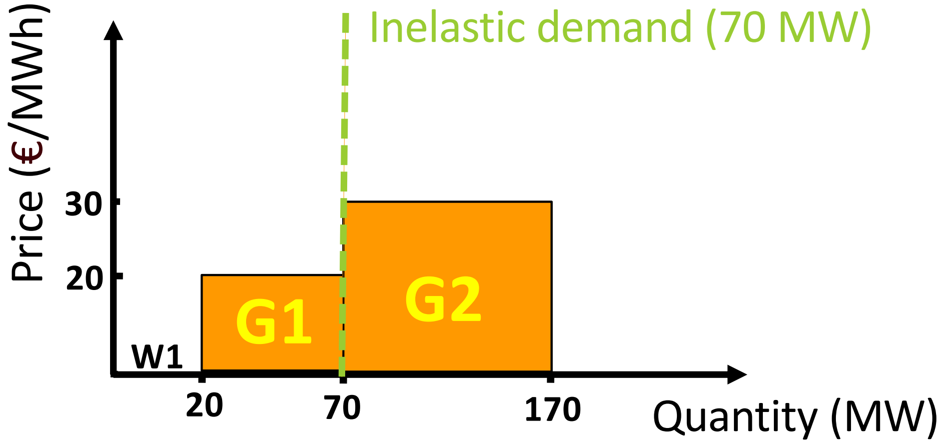
An illustrative example: Different demand

Consider an electricity market consisting of two conventional generators (G1 and G2) and one wind farm (W1), where the total demand is price-inelastic.

Production forecast: 20 MW
Offer price: €0/MWh

Capacity: 50 MW
Offer price: €20/MWh

Capacity: 100 MW
Offer price: €30/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **50**
- Production level (MW) of G2: **0**
- Market-clearing price (€/MWh): **[20,30]**

Price multiplicity: Any price between 20 and 30 (inclusive) can clear the market (verify by the KKT conditions)

An illustrative example: Price-elastic demand

Consider an electricity market consisting of two conventional generators (G1 and G2) and one wind farm (W1). The single demand (D1) is elastic to price.

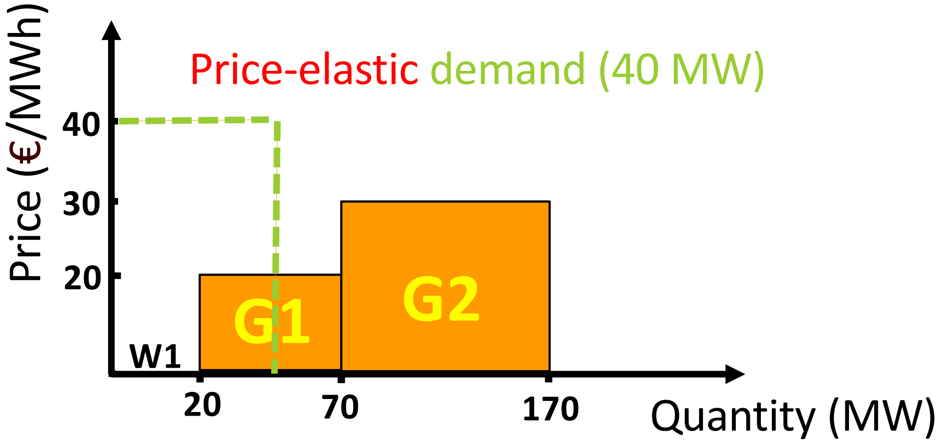
Production forecast: 20 MW
Offer price: €0/MWh

Capacity: 50 MW
Offer price: €20/MWh

Capacity: 100 MW
Offer price: €30/MWh

Demand D1

Maximum load: 40 MW
Bid price: €40/MWh



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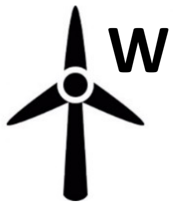


W1

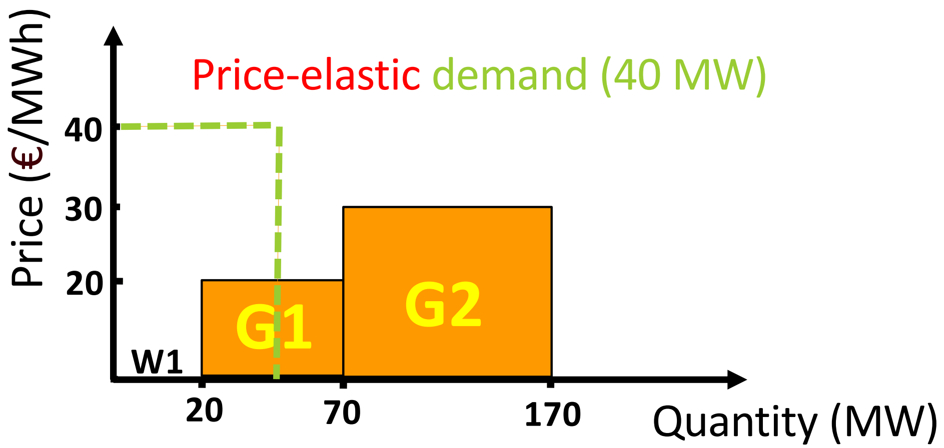
Production forecast: 20 MW
Offer price: €0/MWh

Capacity: 50 MW
Offer price: €20/MWh

Capacity: 100 MW
Offer price: €30/MWh

Demand D1

Maximum load: 40 MW
Bid price: €40/MWh



Market-clearing outcomes:

- Production level (MW) of W1: **20**
- Production level (MW) of G1: **20**
- Production level (MW) of G2: **0**
- Consumption level (MW) of D1: **40**
- Market-clearing price (€/MWh): **20**

Verify by the KKT conditions!

Market clearing with price-elastic demand

$$\underset{\boxed{p^{\mathrm{D1}}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Maximize}} \quad 40\,\boxed{p^{\mathrm{D1}}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}$$

Newly added primal variable: Consumption level of demand D1

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$0 \leq \boxed{p^{\mathrm{D1}}} \leq 40 \quad : \quad \underline{\mu}^{\mathrm{D1}}, \overline{\mu}^{\mathrm{D1}}$$

$$\boxed{p^{\mathrm{D1}}} - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Market clearing with price-elastic demand

$$\underset{p^{\mathrm{D1}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Maximize}} \quad 40\ p^{\mathrm{D1}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$0 \leq p^{\mathrm{D1}} \leq 40 \quad : \quad \boxed{\underline{\mu}^{\mathrm{D1}}, \overline{\mu}^{\mathrm{D1}}}$$ Newly added dual variables

$$p^{\mathrm{D1}} - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Market clearing with price-elastic demand

$$\underset{p^{\mathrm{D1}},p^{\mathrm{W1}},p^{\mathrm{G1}},p^{\mathrm{G2}}}{\text{Maximize}} \quad \underbrace{40\ p^{\mathrm{D1}} - 0\ p^{\mathrm{W1}} - 20\ p^{\mathrm{G1}} - 30\ p^{\mathrm{G2}}}_{\text{Objective function: Social welfare}}$$

subject to:

$$0 \leq p^{\mathrm{W1}} \leq 20 \quad : \quad \underline{\mu}^{\mathrm{W1}}, \overline{\mu}^{\mathrm{W1}}$$

$$0 \leq p^{\mathrm{G1}} \leq 50 \quad : \quad \underline{\mu}^{\mathrm{G1}}, \overline{\mu}^{\mathrm{G1}}$$

$$0 \leq p^{\mathrm{G2}} \leq 100 \quad : \quad \underline{\mu}^{\mathrm{G2}}, \overline{\mu}^{\mathrm{G2}}$$

$$0 \leq p^{\mathrm{D1}} \leq 40 \quad : \quad \underline{\mu}^{\mathrm{D1}}, \overline{\mu}^{\mathrm{D1}}$$

$$p^{\mathrm{D1}} - p^{\mathrm{W1}} - p^{\mathrm{G1}} - p^{\mathrm{G2}} = 0 \quad : \quad \lambda$$

Market clearing as a linear optimization problem

Compact form:

Market clearing as a linear optimization problem

Compact form:

$$\underset{p_g^{\mathrm{G}},p_d^{\mathrm{D}}}{\text{Maximize}} \quad SW = \sum_d U_d \; p_d^{\mathrm{D}} - \sum_g C_g \; p_g^{\mathrm{G}} \qquad \text{(1a)}$$

subject to:

$$0 \leq p_d^{\mathrm{D}} \leq \overline{P}_d^{\mathrm{D}} \quad \forall d \qquad \text{(1b)}$$

$$0 \leq p_g^{\mathrm{G}} \leq \overline{P}_g^{\mathrm{G}} \quad \forall g \qquad \text{(1c)}$$

$$\sum_d p_d^{\mathrm{D}} - \sum_g p_g^{\mathrm{G}} = 0 \qquad : \lambda \qquad \text{(1d)}$$

U_d : bid price of demand d

C_g : offer price of generator g

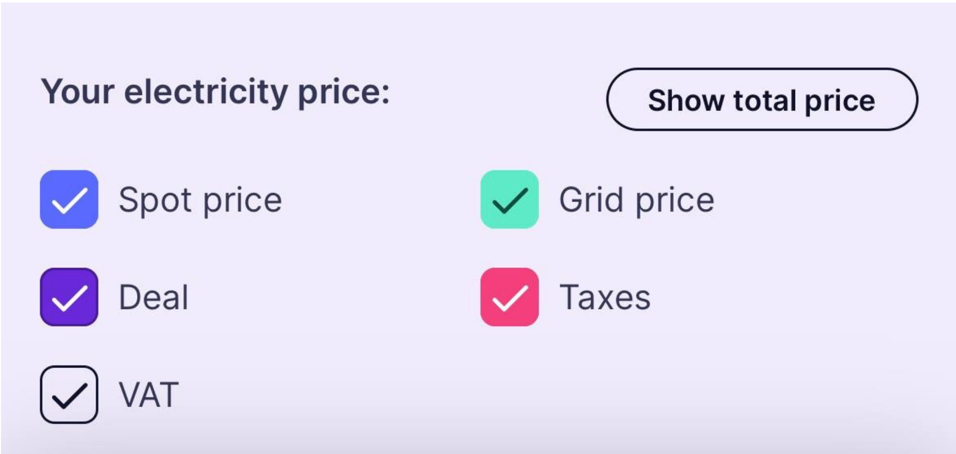
$\overline{P}_d^{\mathrm{D}}$: maximum load of demand d

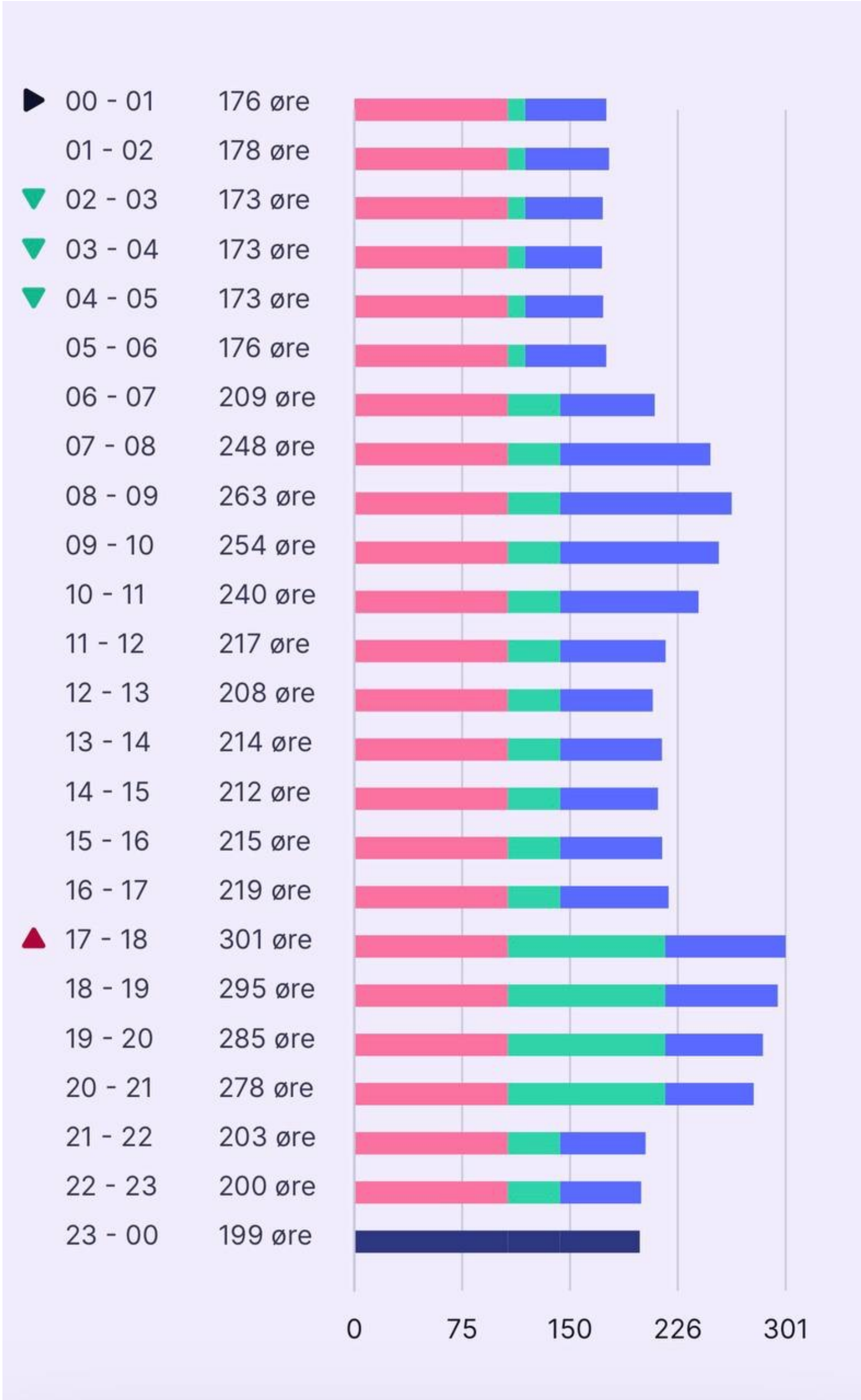
$\overline{P}_g^{\mathrm{G}}$: capacity of generator g

Electricity price in DK

What is the typical electricity price for residential consumers in Denmark? What are the components of the rate?

Prices for Jalal’s place, February 10, 2025
Source: Elekt [link]



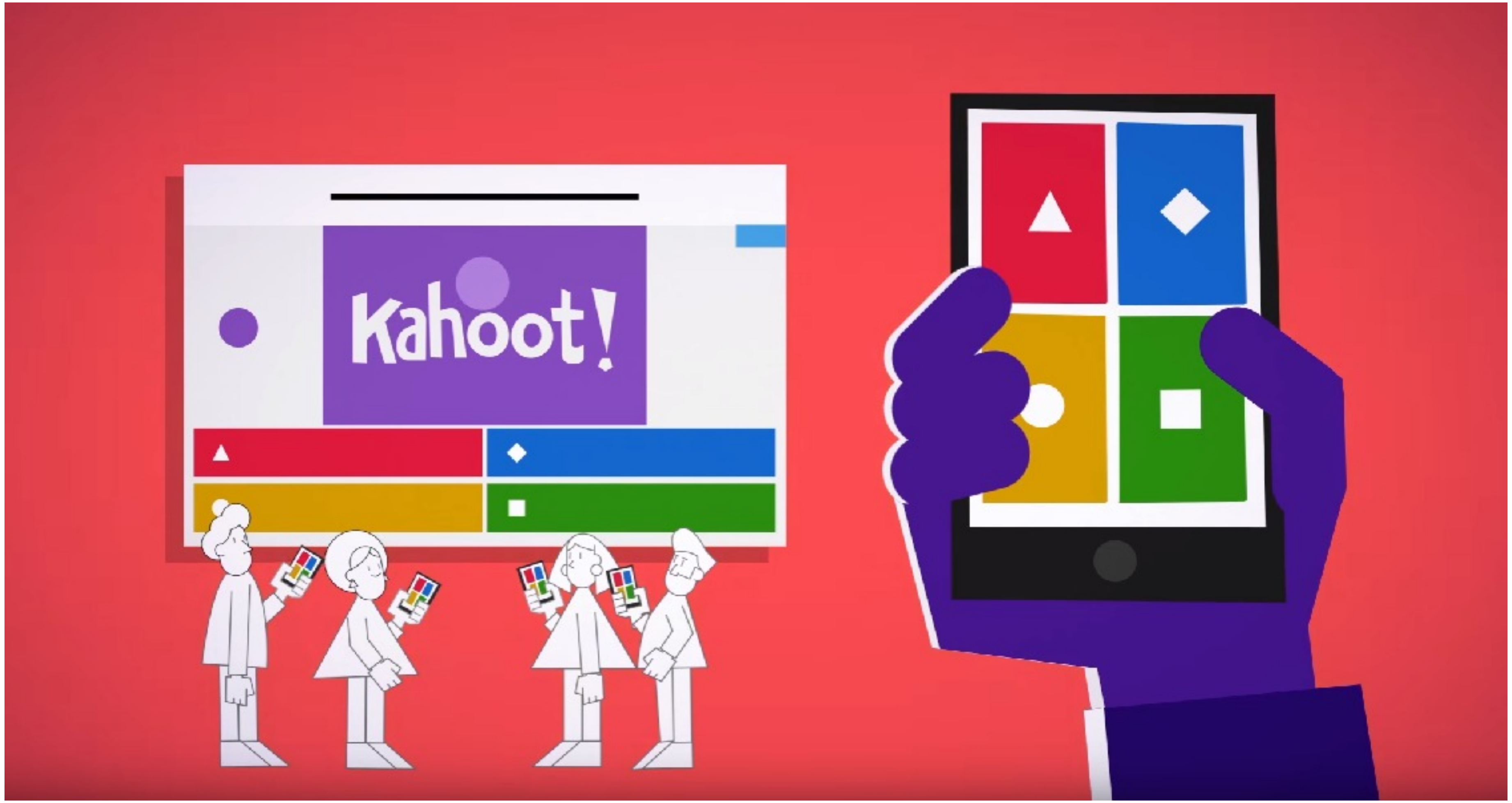


Assignment 1

Full assignment is uploaded on DTU Learn and my webpage:
https://www.jalalkazempour.com/teaching

Your tasks:

- Select a case study and complete it by finding data from the data sources introduced.
- **Step 1: Copper-Plate, Single Hour** (based on today's lecture, i.e., Lecture 2)
- **Step 2: Copper-Plate, Multiple Hours** (you can start working on it based on today's lecture)
- **Step 3: Network Constraints** (you can work on it after Lecture 3)
- **Step 4: Optimization vs. Equilibrium** (you can work on it after Lecture 4)
- **Step 5: Balancing Market** (you can work on it after Lecture 5)
- **Step 6: Reserve Market** (you can work on it after Lecture 6)


Kahoot!

Thanks for your attention!

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